Online Motion Planning MA-INF 1314

Summersemester 17

Escape Paths

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Escape Path situation

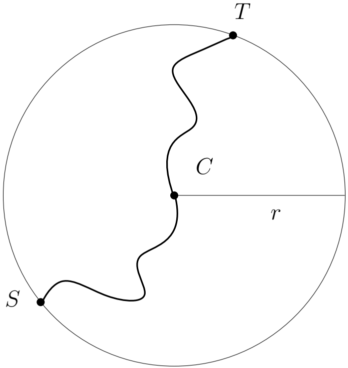
- Try to *escape* from an partially unknown environment
- The adversary manipulates the environment
- Leave the area as soon as possible
- *Lost in a forest* Bellman 1956
- *Escape paths* for known region R
- Single deterministic path
- Leave area from any starting point
- Adversary translates and rotates R
- Minimize the length of successful path
- Geometric argumentations
- Only known for few shapes

Simple examples

Obviously: The diameter of any region R is always an escape path!

Theorem: The shortest escape path for a circle of radius r is a line segment of length 2r.

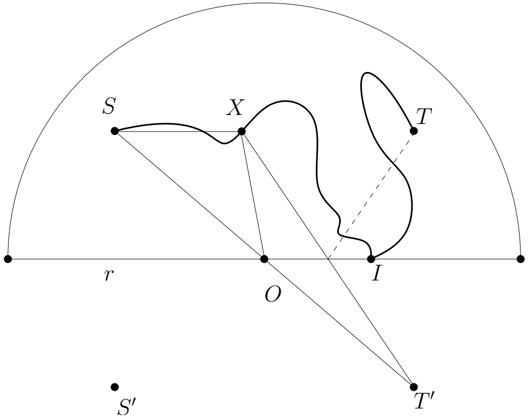
Proof: Assume there is a better escape path! Contradiction!



Also for semicircles

Theorem: The shortest escape path for a semicircle of radius *r* is a line segment of length 2r.

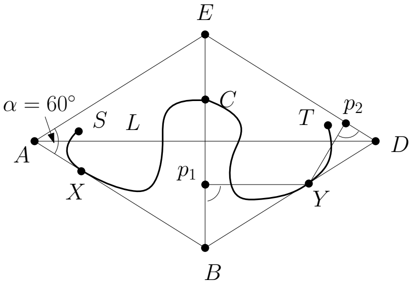
Proof: Assume there is a better escape path! Contradiction!



More generally for a rhombus with angle 60°

Theorem: The shortest escape path for a rhombus of diameter *L* with angle $\alpha = 60^\circ$ is a line segment of length *L*.

Proof: Assume there is a better escape path! Contradiction!

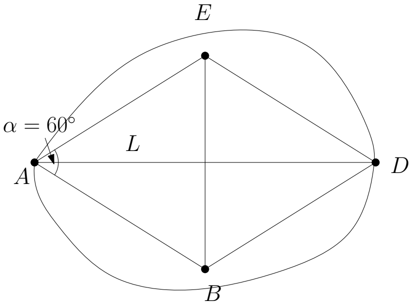


Fatness definition!

Definition: Fatness w.r.t. diameter! Rhombus-Fat!

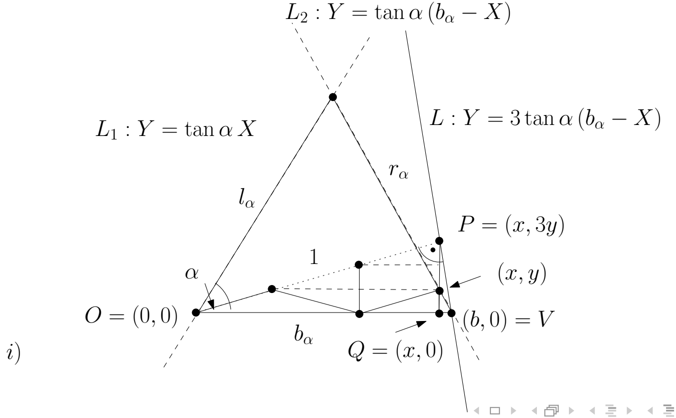
Corollary: The shortest escape path for rhombus-fat convex set of diameter L is a line segment of length L.

Proof: Assume there is a better escape path! Contradiction!

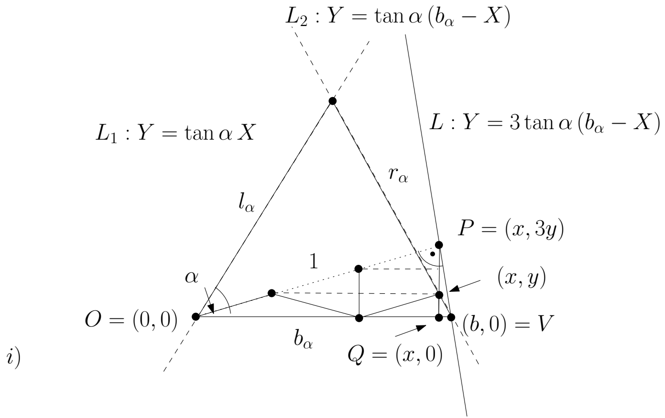


Convex = diameter?

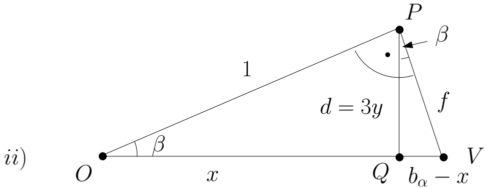
- Equilateral triangle: Besicovitch
- Zig-Zag escape path with length ≈ 0.9812
- More generally from Coulton and Movshovich (2006)
- Isosceles triangle for α and b_α
- b_α is diameter!



- Construct symmetric Zig-Zag path of small length
- Asssume length 1.



- Extract triangle
- $\frac{1}{x} = \frac{b_\alpha}{1}$ $x = \frac{1}{b_\alpha}$

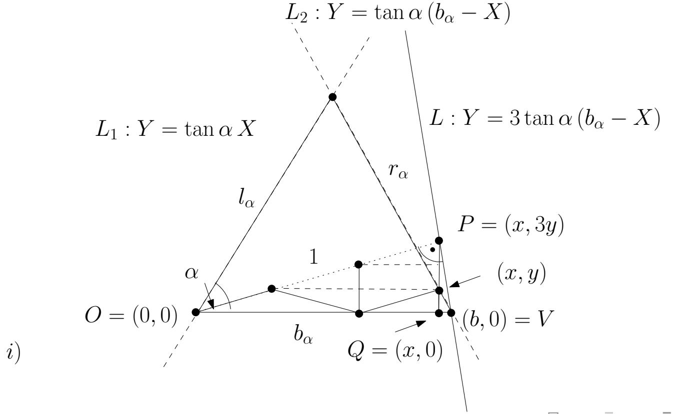


Convex = diameter?

Finally we determine b_α:

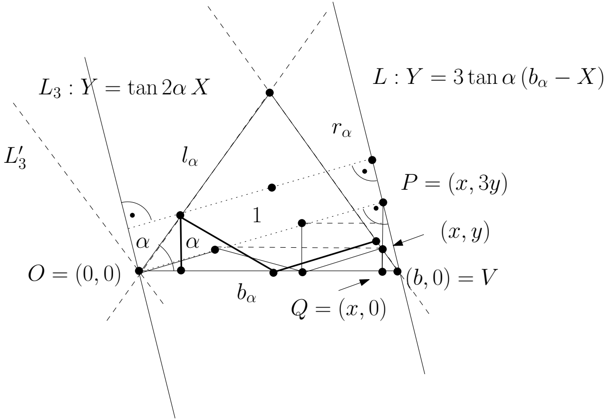
$y = \tan\alpha \left(b_\alpha - \frac{1}{b_\alpha}\right)$ and $x = \frac{1}{b_\alpha}$ and $x^2 + (3y)^2 = 1$ which gives

$$b_\alpha = \sqrt{1 + \frac{1}{9\tan^2\alpha}}\,.$$



Further constraint for α

There should be no better Zig-Zag path for T_α!
Line $L_3 : Y = \tan(2\alpha)$ runs in parallel with L_2. This means $-3\tan\alpha = \tan 2\alpha$ or $\tan\alpha = \sqrt{\frac{5}{3}}$.



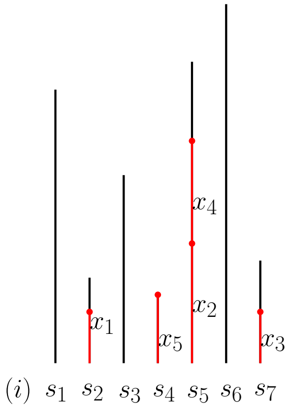
Besicovitch triangles

Theorem: For any $\alpha \in [\arctan(\sqrt{\frac{5}{3}}), 60^\circ]$ there is a symmetric Zig-Zag path of lenght 1 that is an escape path of T_α smaller than the diameter b_α.

- $b_\alpha = \sqrt{1 + \frac{1}{9\tan^2\alpha}}$
- $\alpha = 60^\circ$: $b_\alpha = \sqrt{\frac{28}{27}}$
- $b_\alpha := 1 \Longrightarrow \sqrt{\frac{27}{28}} < 1$ is Zig-Zag path length
- Optimality? Yes!

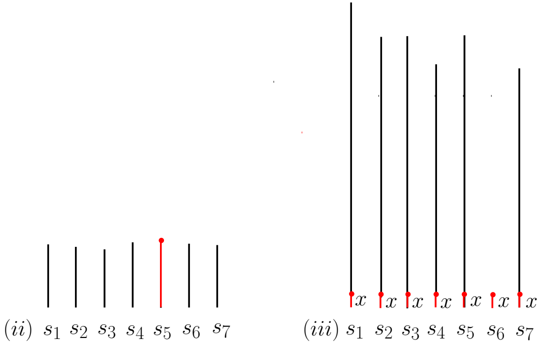
Different performance measures

- Set L_m of m line segments s_i of unknown length $|s_i|$
- Dark corridors, escape, digging for oil
- Test corridors successively
- s_{j_1} up to a certain distance x_1, then s_{j_2} for another distance x_2 and so on

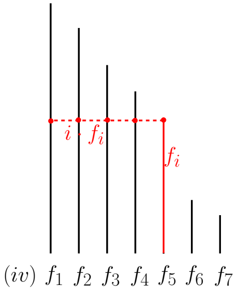


More information

- Assume distribution is known!
- $f_1 \geq f_2 \geq \cdots \geq f_m$ order of the length given
- Extreme cases! Good strategies!

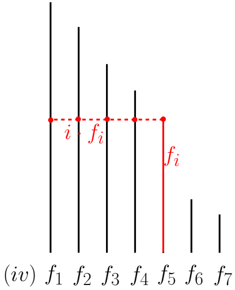


- $f_1 \geq f_2 \geq \cdots \geq f_m$ order of the length given
- Check i arbitrary segments with length f_i:
  $\min_i i \cdot f_i$ is the best strategy



Known length in general

- $f_1 \geq f_2 \geq \cdots \geq f_m$ order of the length given
- Check i arbitrary segments with length f_i:
  $\min_i i \cdot f_i$ is a reasonable strategy
- $C(F_m, A)$ travel cost for algorithm A
- $\text{maxTrav}(F_m) := \min_A C(F_m, A)$

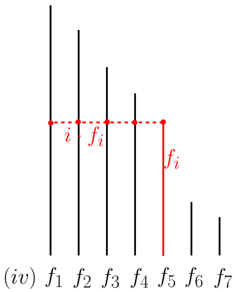


Optimal strategy for this case

Theorem: For a set of sorted distances F_m (i.e. $f_1 \geq f_2 \geq \cdots \geq f_m$) we have

$$\mathrm{maxTrav}(F_m) := \min_i i \cdot f_i \,.$$

Proof:



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Proof:

- Arbitrary strategy A
- Less than $\min_i i \cdot f_i$ means less than $j \cdot f_j$ for any j
- Visiting depth $d_1 \geq d_2 \geq \cdots \geq d_m$
- Not reached f_1 by d_1,
  not reached f_2, since $d_1 + d_2 < 2f_2$ and $d_2 \leq d_1$ and so on
- Not successful!
- $\min_i i \cdot f_i$ always sufficient!

Online Strategy

- F_m with $f_1 \geq f_2 \geq \cdots \geq f_m$ not known
- Compete against $\text{maxTrav}(F_m) := \min_i i \cdot f_i$
- Dovetailing strategy: Rounds $c = 1, 2, 3, 4, \ldots$
- For any round c from left to right:
  Path length of segment i is *extended* up to distance $\left\lfloor \frac{c}{i} \right\rfloor$

Online Strategy

- Dovetailing strategy: Rounds $c = 1, 2, 3, 4, \ldots$
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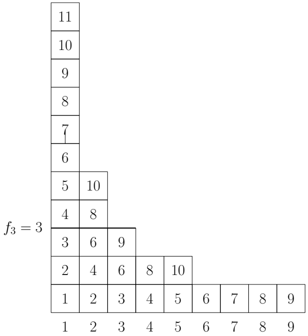


Online Strategy!

Theorem: Hyperbolic traversal algorithm solves the multi-segment escape problem for any list F_m with maximum traversal cost bounded by

$$D \cdot (\text{maxTrav}(F_m) \ln(\min(m, \text{maxTrav}(F_m)))$$

for some constant D.

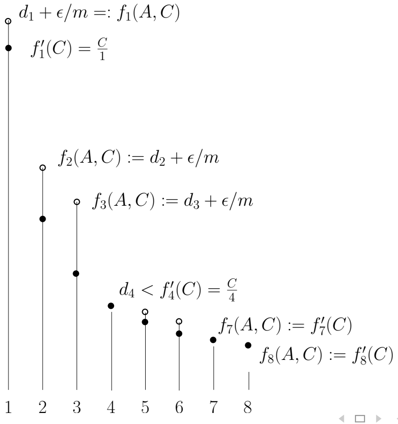
Proof:(W.l.o.g. F_m integers)

- Let $\min_i i \cdot f_i = j \cdot f_j$ for some j
- c with $c = j \cdot f_j$ exists (Round c)
- Overoll cost:

$$\sum_{t=1}^{m} \left\lfloor \frac{c}{t} \right\rfloor \leq \sum_{t=1}^{\min(m,c)} \frac{c}{t} \leq c + \int_1^{\min(m,c)} \frac{c}{t}\, dt = c(1 + \ln \min(m, c))\,.$$

Matches Lower bound!

Theorem: For any deterministic online strategy A that solves the multi-segment escape problem we can construct input sequences $F_m(A, C)$ so that A has cost at least $d \cdot C \ln \min(C, m)$ and $\text{maxTrav}(F_m(C, A)) \leq C$ holds for some constant d and arbitrarily large values C.



Matches Lower bound! Proof!

- C is given! $f_i'(C) = \frac{C}{i}$ (not yet fixed)
- Wait until cost $\sum_{i=1}^{m} d_i \geq d \cdot C \ln \min(C, m)$ for some d
- Fix the scenario as shown below!