Online Motion Planning MA-INF 1314
Searching in streets!

Elmar Langetepe
University of Bonn

Rep: Summary searching for rays

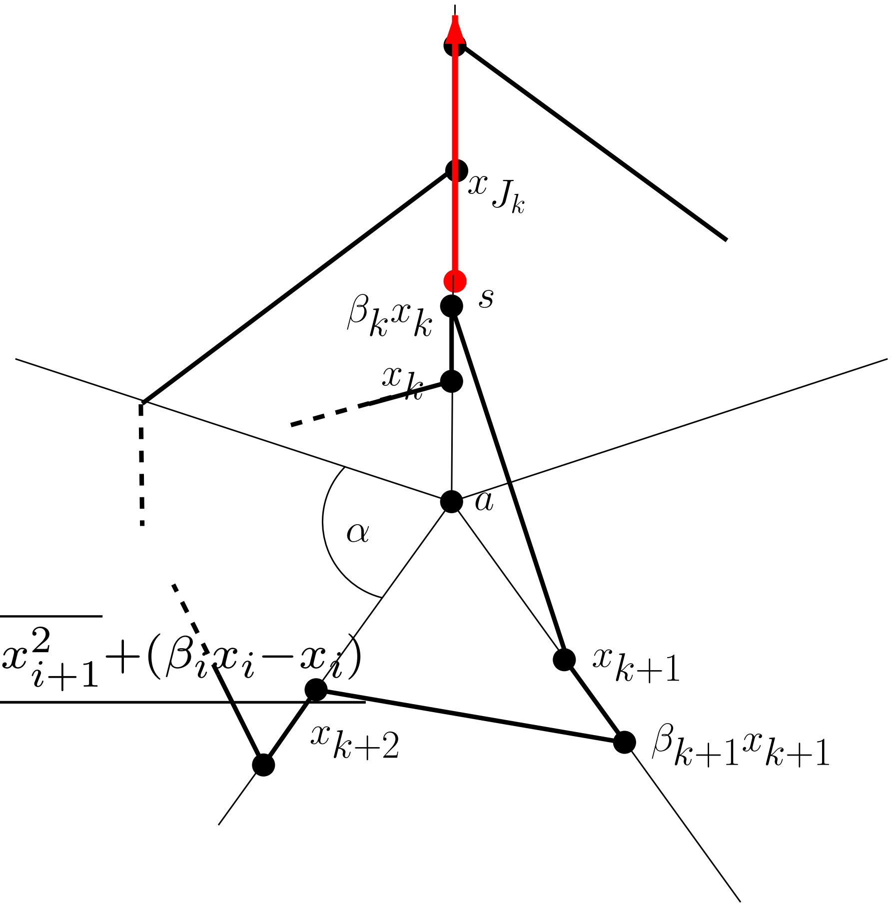
- The Window-Shopper-Problem
- Optimal strategy $C = 1.059\ldots$: **Theorem**
- Interesting design technique
- Rays in general
- Lower $C \geq 2\pi\, e = 17.079\ldots$ (**Theorem**) and upper bound $C = 22.51\ldots$ (**Theorem**)
- Lower bound construction
- Also a lower bound for special case with $C = 17.289\ldots$

Rep.: Lower bound construction, special rays

- Find s on a ray visited up to $\beta_k x_k$ at the last time, now at x_{J_k}
- Note: Any order is possible
- Worst-case, s close to $\beta_k x_k$
- Ratio: $C(S)$

$$\frac{\sum_{i=1}^{J_k-1} \sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)}{\beta_k x_k}$$

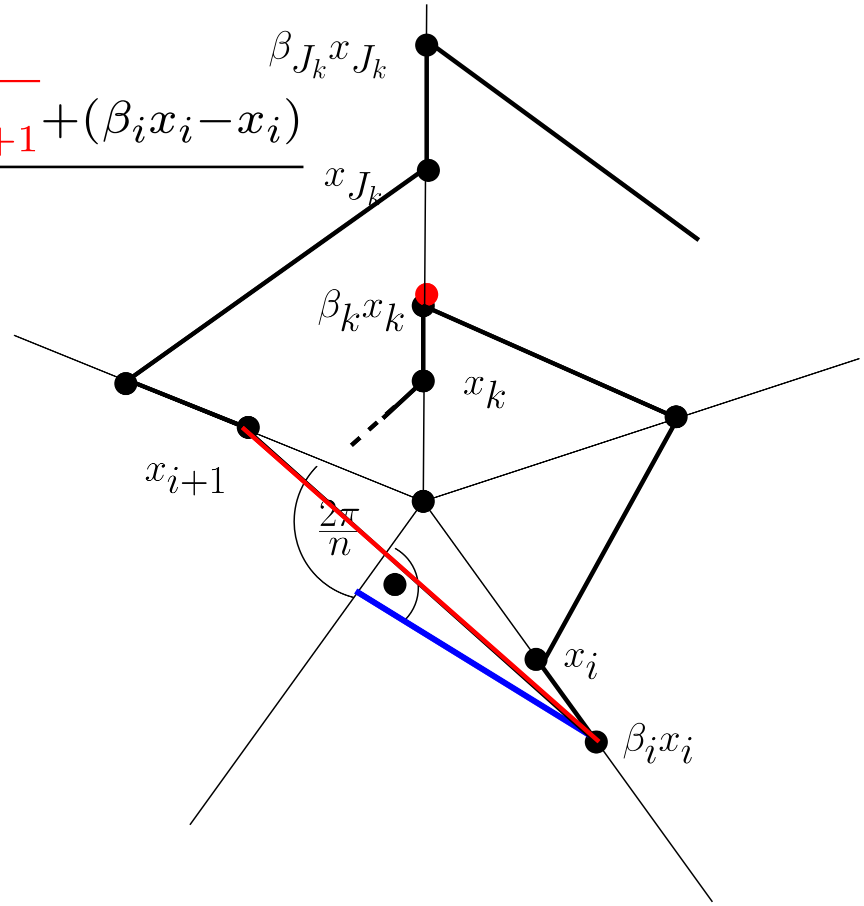
- Monotone/Periodic, Functional??



Rep.: Lower bound construction, special rays

- Ratio: $C(S)$

$$\frac{\sum_{i=1}^{J_k-1} \sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)}{\beta_k x_k}$$



- Shortest distance to next ray: $\beta_i x_i \sin \frac{2\pi}{n}$
- Lower bound for

$$\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2}$$

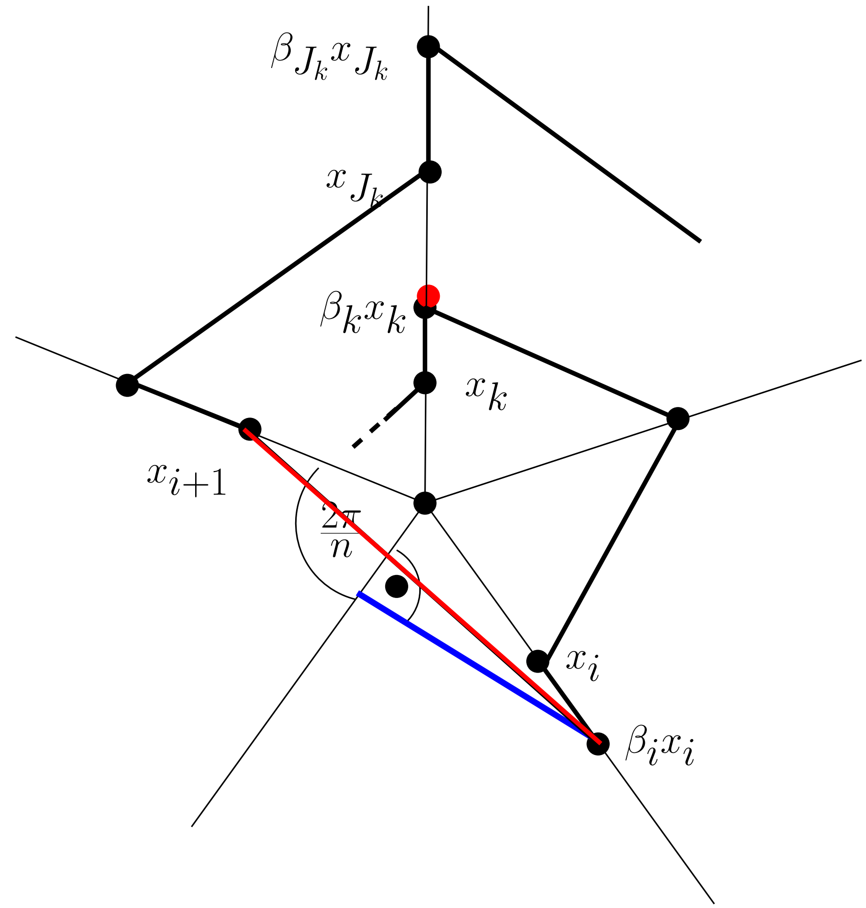
- Lower bound: $C(S) \geq$

$$\sin \frac{2\pi}{n} \frac{\sum_{i=1}^{J_k-1} \beta_i x_i}{\beta_k x_k}$$

Rep.: Lower bound construction

- Lower bound $\frac{\sum_{i=1}^{J_k-1} f_i}{f_k}$
- Equals functional of standard m-ray search
- Optimal strategy: monotone/periodic (Alpern/Gal)
- $f_i = \left(\frac{n}{n-1}\right)^i$,

  ratio: $(n-1)\left(\frac{n}{n-1}\right)^n$
- $C(S) \geq \sin\frac{2\pi}{n}(n-1)\left(\frac{n}{n-1}\right)^n$

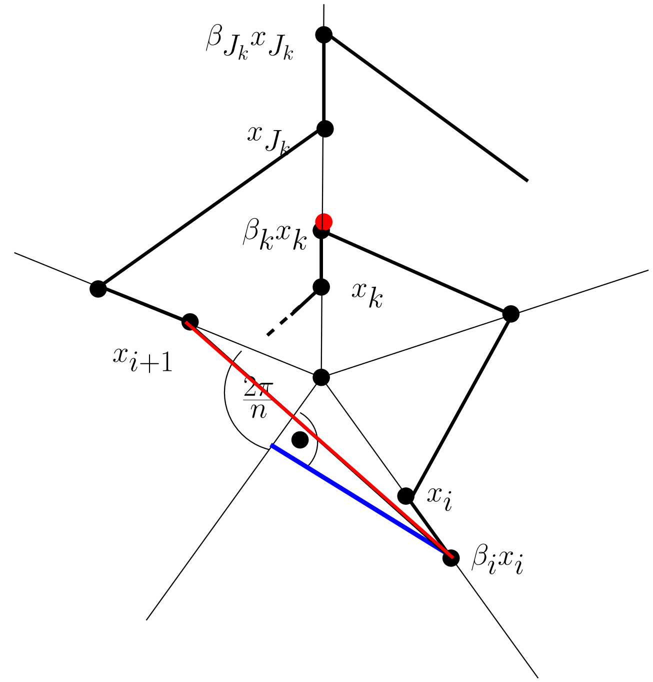


Rep.: Lower bound construction

- $C(S) \geq \sin\frac{2\pi}{n}(n-1)\left(\frac{n}{n-1}\right)^n$

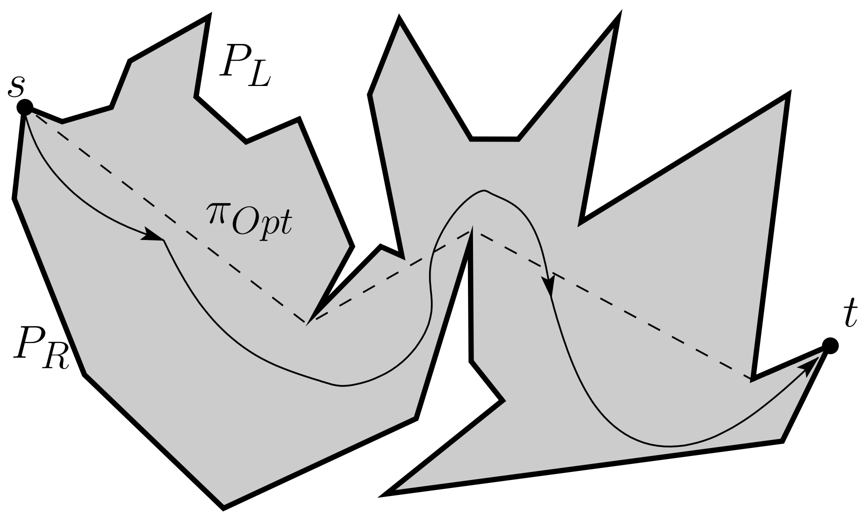
$$\lim_{n\to\infty}(n-1)\left(\frac{n}{n-1}\right)^n \sin\frac{2\pi}{n} =$$
$$2\pi\, e = 17.079\ldots$$

- Lower bound: **Theorem**



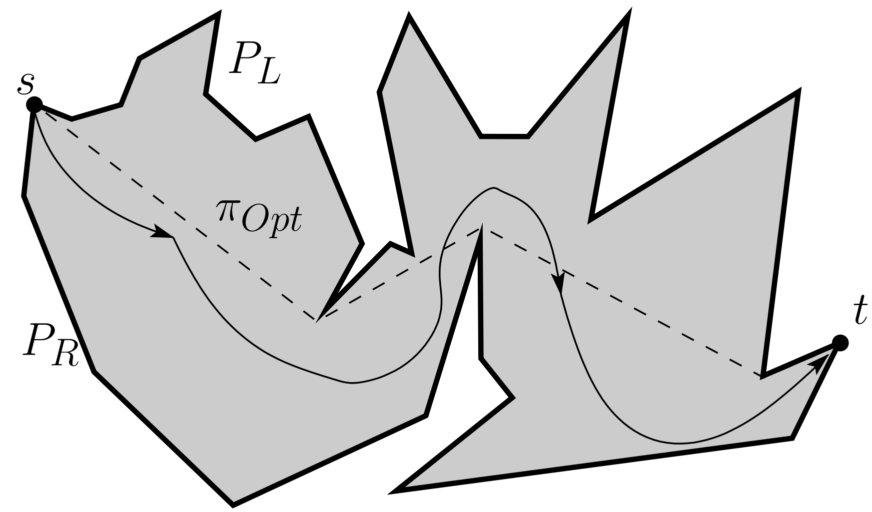
Searching in street polygons

- Searching in simple ploygons, visibility
- Subclass: Streets
- Start- and target
- Target t unknown, search for t!
- Compare to shortest path to t! Comp. factor!!



Formal definition

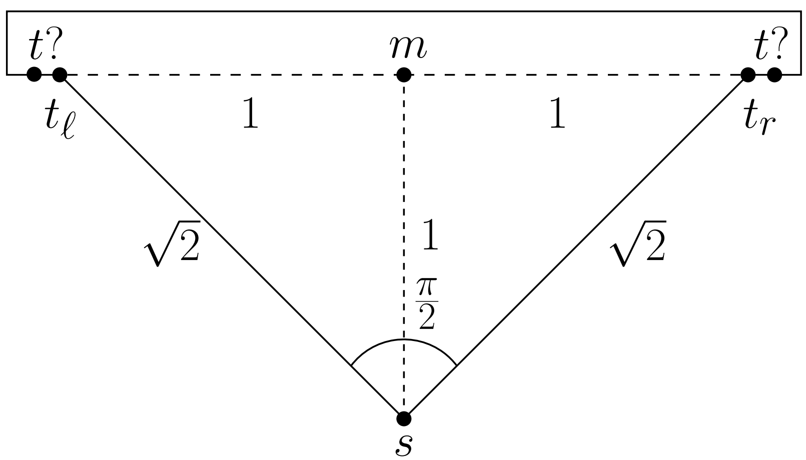
Def. Let P be a simple polygon with t and s on the boundary of P. Let P_L und P_R denote the left and right boundary chain from s to t. P is denoted as a *street*, if P_L and P_R are *weakly visible*, i.e., for any point $p \in P_L$ there is at least one point $q \in P_R$ that is visible, and vice versa.



Lower Bound

Theorem No strategy can achieve a path length smaller than $\sqrt{2} \times \pi_{\text{Opt}}$.

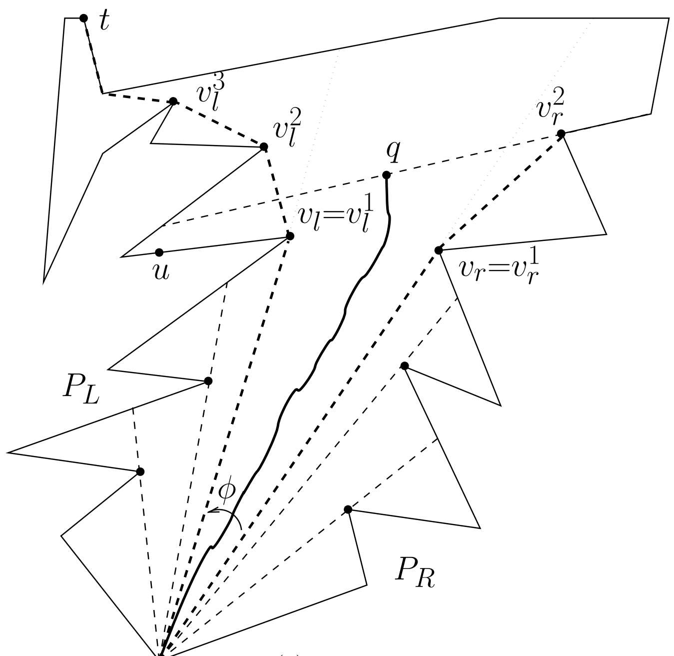
Proof:



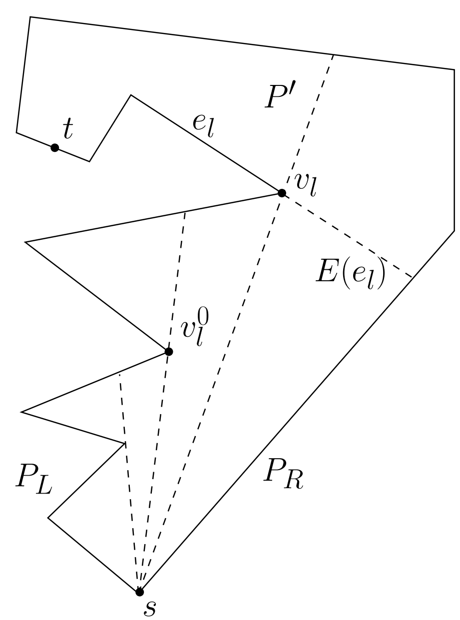
Detour with ratio $\sqrt{2}$

Reasonable movements: Struktural property!

- Inner wedge is important: Between ...
- Rightmost left reflex vertex, leftmost right reflex vertex
- By contradiction: Assume that the goal is not there! No street!



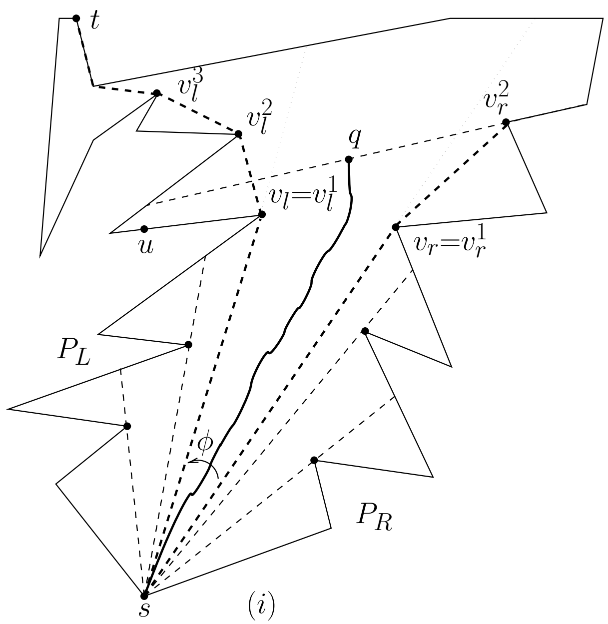
(i)

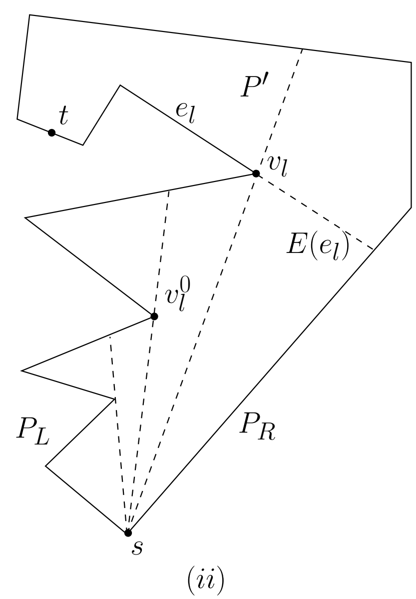


(ii)

Reasonable strategies

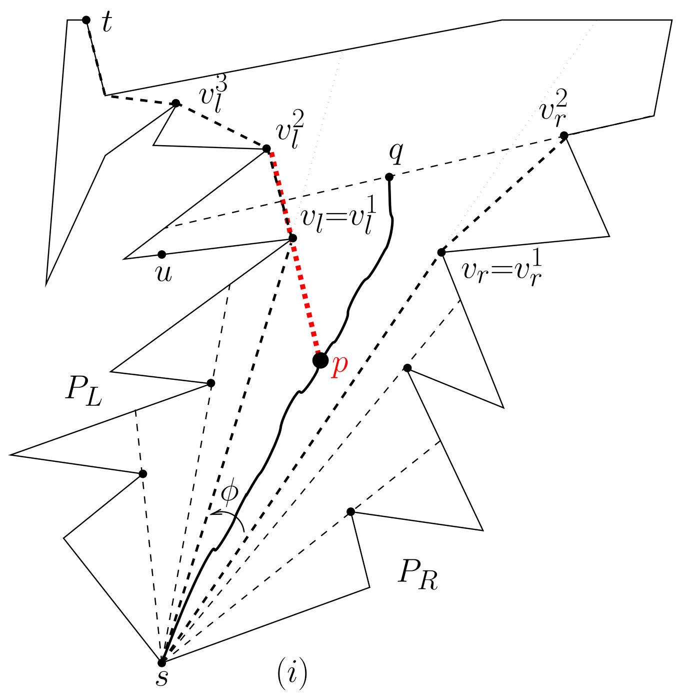
- Always into wedge of c, v_l and v_r
- Goal visible, move directly toward it
- Cave behind v_ℓ or v_r fully visible, no target as for q (v_ℓ or v_r vanishes), agent moves directly to the opposite vertex





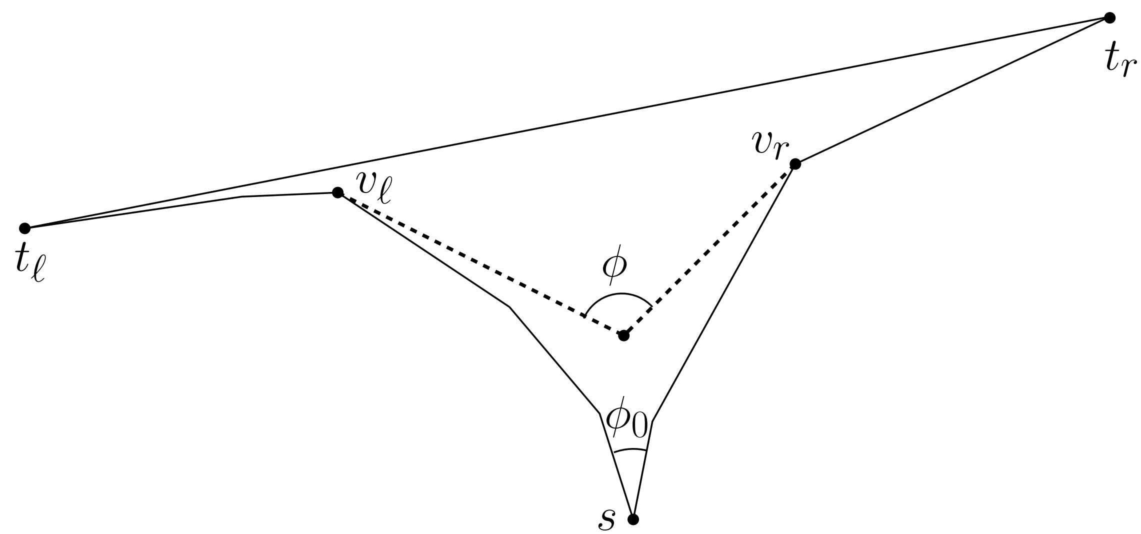
Reasonable strategies

- Always into wedge of c, v_l and v_r
- Another vertex (for example) v_ℓ^2 appears behind v_ℓ.
  Change to the wedge c, v_l^2 and v_r



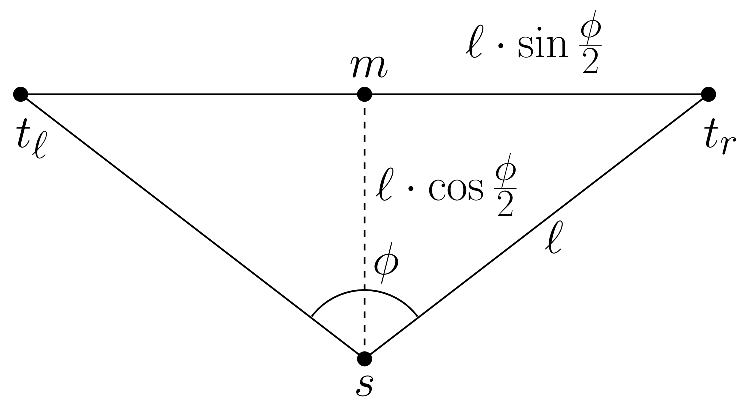
Funnel situation!

- It is sufficient to consider special streets only!
- Combine them piecewise!
- **Def.** A polygon that start with a convex vertex s and consists of two opening convex chains ending at t_ℓ and t_r respectively and which are finally connected by a line segment $\overline{t_\ell t_r}$ is called a *funnel* (polygon).



Generalized Lower Bound

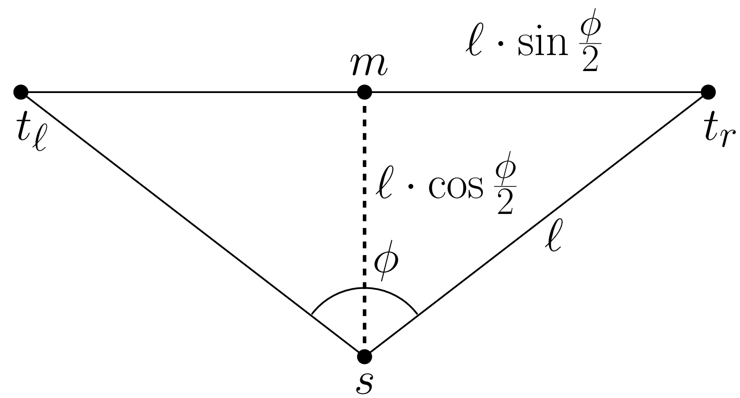
Lemma For a funnel with opening angle $\phi \leq \pi$ no strategy can guarantee a path length smaller than $K_\phi \cdot |Opt|$ where $K_\phi := \sqrt{1 + \sin\phi}$. Proof:



Detour at least: $\frac{|\pi_S|}{|\pi_{Opt}|} = \frac{\ell\cos\frac{\phi}{2} + \ell\sin\frac{\phi}{2}}{\ell} = \sqrt{1 + \sin\phi}$.

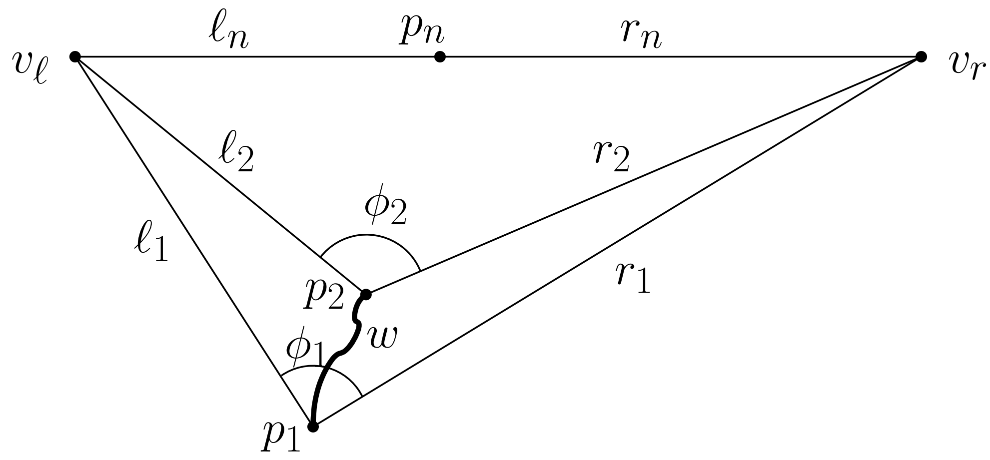
Opt. strat. opening angle $0 \leq \varphi_0 \leq \pi$!

- $K_\phi := \sqrt{1 + \sin\phi}$.
- Strongly increasing: $0 \leq \phi \leq \pi/2$, Interval $[1, \sqrt{2}]$
- Strongly decreasing: $\pi/2 \leq \phi \leq \pi$, Interval $[\sqrt{2}, 1]$
- Subdivide: Strategy up to $\phi_0 = \pi/2$, Strategy from $\phi_0 = \pi/2$
- Here: Start from s with angle $\phi_0 \geq \pi/2$.
- Remaining case: Exercise!



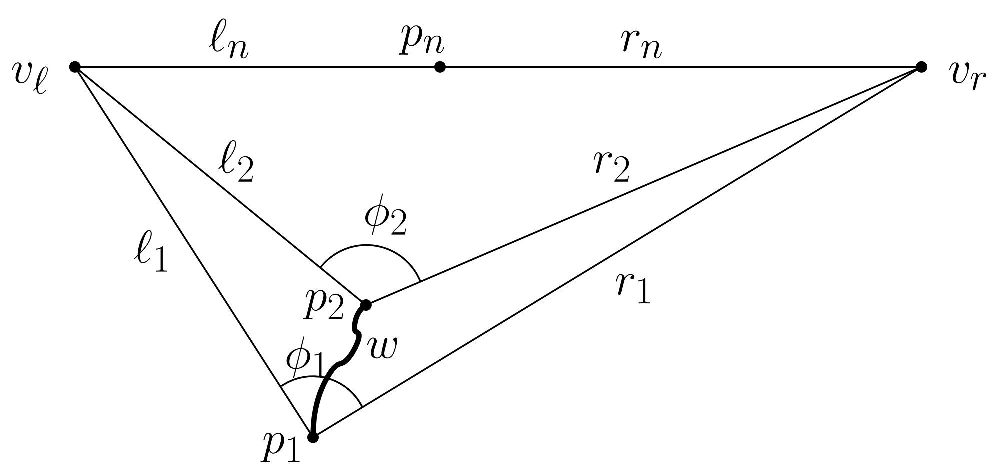
Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

- Backward analysis: For $\varphi_n := \pi$ optimal strategy.
- $K_\pi = 1$ and K_π-competitive opt. strategy with path l_n or r_n!
- Assumption: Opt. strategy for some ϕ_2 with factor K_{ϕ_2} ex.
- How to prolong for ϕ_1 with factor K_{ϕ_1} where $\frac{\pi}{2} \leq \phi_1 < \phi_2$?
- We have $K_{\phi_1} > K_{\phi_2}$



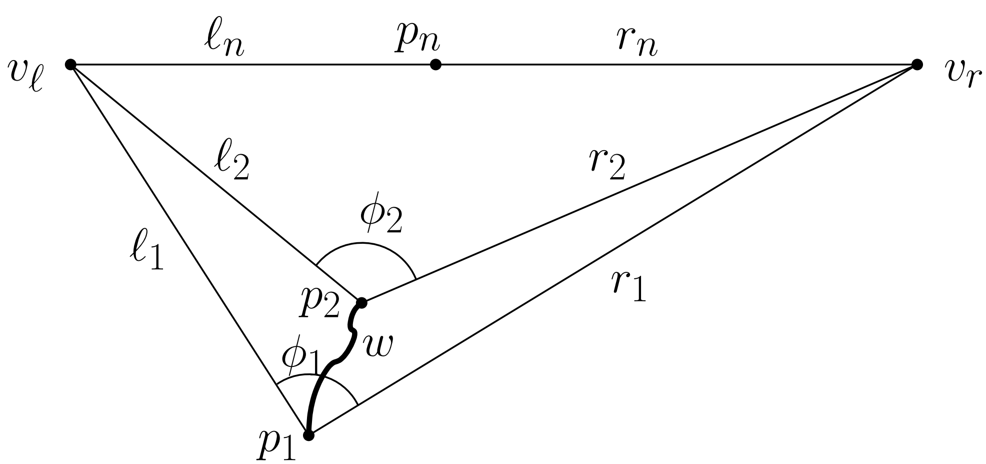
Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

- Situation: Opt. strategy for ϕ_2 with ratio K_{ϕ_2}
- How to get opt. strategy for K_{ϕ_1}?
- Conditions for the path w? Design!
- Goal behing v_l, path: $|w| + K_{\phi_2} \cdot \ell_2$, optimal: l_1
- Goal behind v_r, path: $|w| + K_{\phi_2} \cdot r_2$, optimal: r_1
- Means: $\frac{|w|+K_{\phi_2} \cdot \ell_2}{l_1} \leq K_{\phi_1}$ and $\frac{|w|+K_{\phi_2} \cdot r_2}{r_1} \leq K_{\phi_1}$



Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

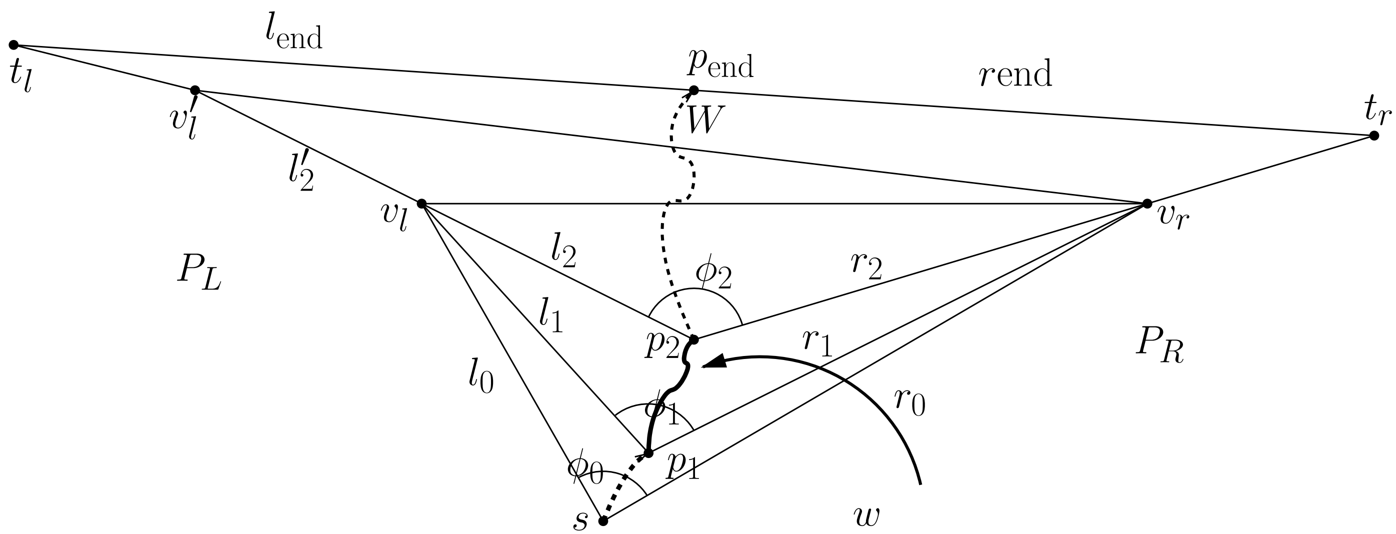
- Guarantee: $\frac{|w|+K_{\phi_2}\cdot \ell_2}{l_1} \leq K_{\phi_1}$ and $\frac{|w|+K_{\phi_2}\cdot r_2}{r_1} \leq K_{\phi_1}$
- Combine, single condition for w
- $|w| \leq \min\{ K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \ , \ K_{\phi_1}r_1 - K_{\phi_2}r_2 \}$
- Change of a vertex at p_2? Remains guilty!



Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

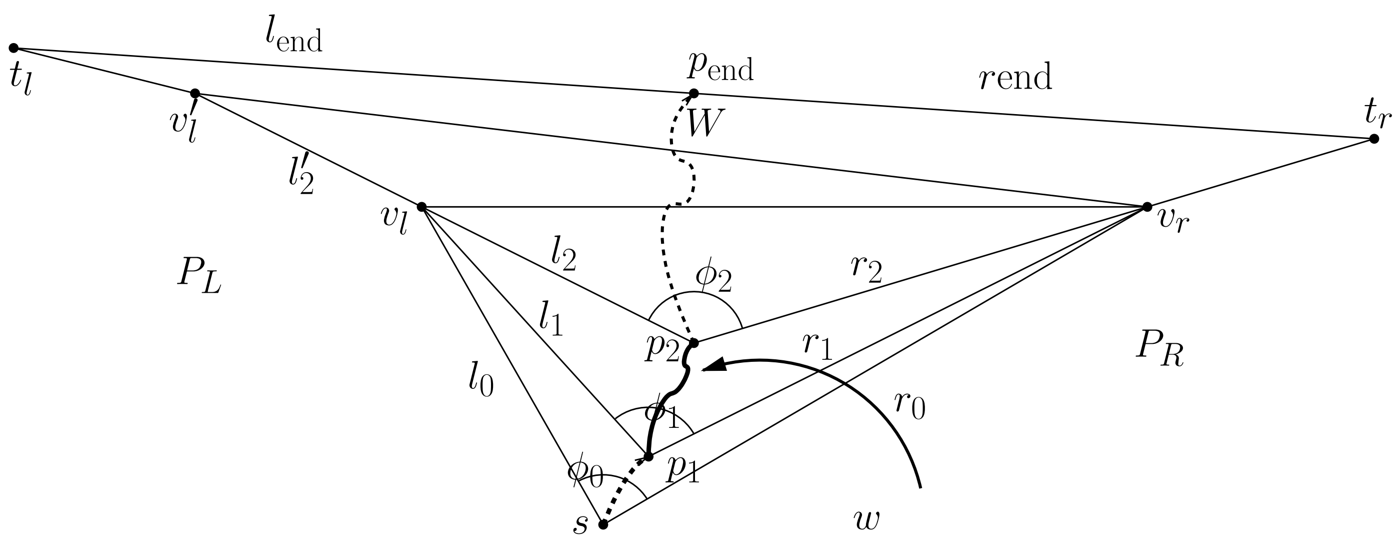
- Change left hand: Condition

  $|w| \leq \min\{\, K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \;,\; K_{\phi_1}r_1 - K_{\phi_2}r_2 \,\}$
- There is opt. strategy for ϕ_2
- Show: $\frac{|w|+K_{\phi_2}\cdot(\ell_2+\ell_2')}{(l_1+\ell_2')} \leq K_{\phi_1}$



Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

$$\begin{aligned} |w| &\leq K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \\ &= K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 + K_{\phi_2}\ell_2' - K_{\phi_2}\ell_2' \\ &\leq K_{\phi_1}(\ell_1 + \ell_2') - K_{\phi_2}(\ell_2 + \ell_2') \end{aligned}$$

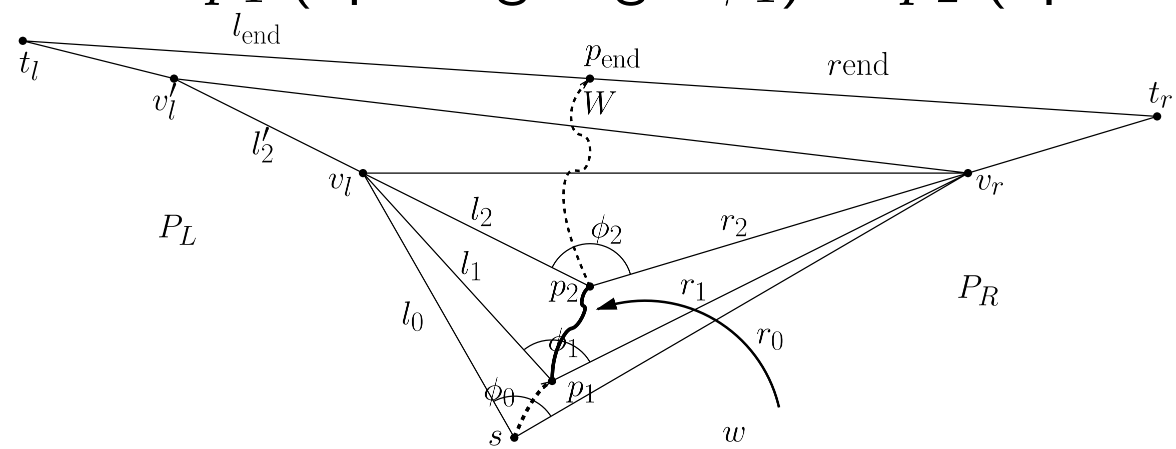


Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

Lemma Let S be a strategy for funnels with opening angles $\phi_2 \geq \frac{\pi}{2}$ and competitive ratio K_{ϕ_2}. We can extend this strategy to a strategy with ratio K_{ϕ_1} for funnels with opening angles ϕ_1 where $\phi_2 > \phi_1 \geq \frac{\pi}{2}$, if we guarantee

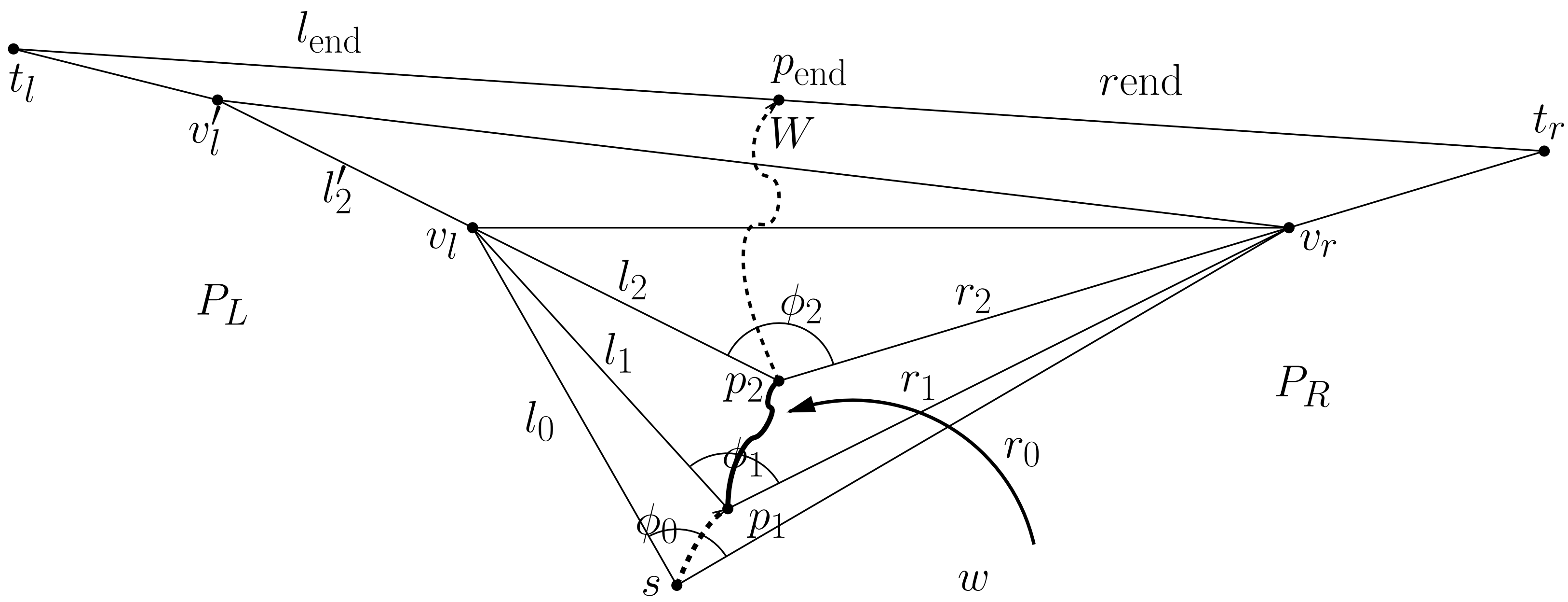
$$|w| \leq \min\{\, K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \,,\; K_{\phi_1}r_1 - K_{\phi_2}r_2 \,\}$$

for the path w from p_1 (opening angle ϕ_1) to p_2 (opening angle ϕ_2).



Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

- If $|w| \leq \min\{\, K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2\;,\; K_{\phi_1}r_1 - K_{\phi_2}r_2 \,\}$ holds, then
- $|W| \leq \min\{\, K_{\phi_0} \cdot |P_L| - K_\pi \ell_{\text{End}}\;,\; K_{\phi_0} \cdot |P_R| - K_\pi r_{\text{End}} \,\}$.

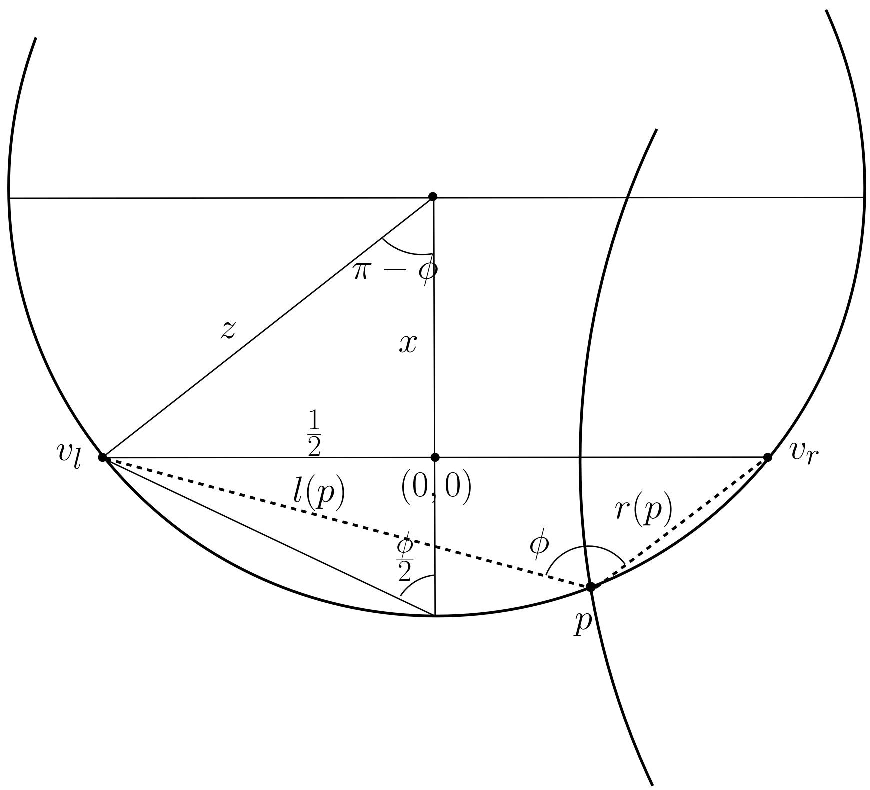


Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2$!

- $|w| \leq \min\{ K_{\phi_1}\ell_1 - K_{\phi_2}\ell_2 \,,\, K_{\phi_1}r_1 - K_{\phi_2}r_2 \}$
- How to fulfil this?
- Equality for both sides: $K_{\phi_2}(\ell_2 - r_2) = K_{\phi_1}(\ell_1 - r_1)$
- Good choice for both sides!
- Defines a curve!
- We start with $A = K_{\phi_0}(\ell_0 - r_0)$
- Parametrisation!

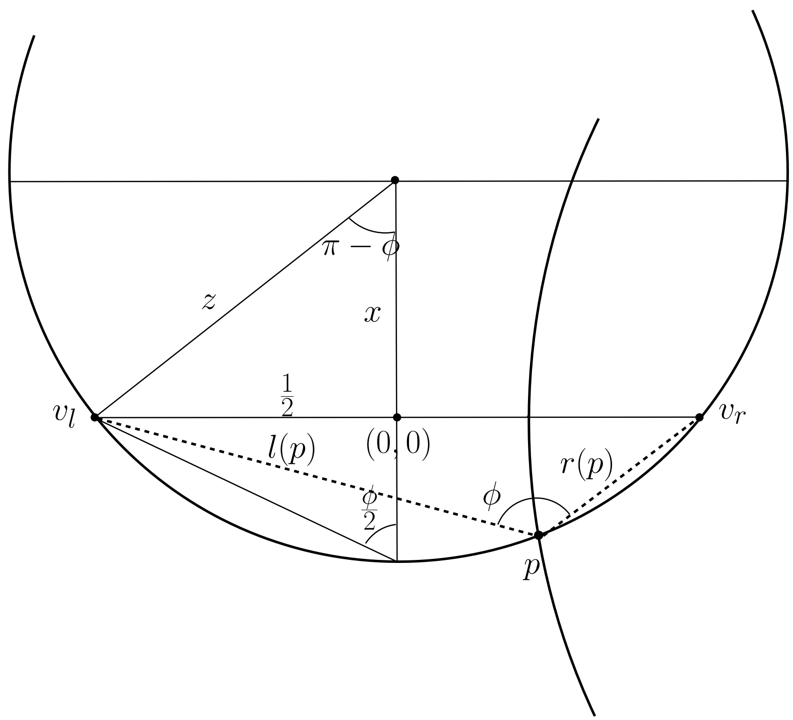
$$A = K_{\phi_0}(\ell_0 - r_0)$$

- Hyperbola: $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$, $l - r = 2a$, $2c$, $a^2 + b^2 = c^2$
- Circle: $X^2 + (Y - x)^2 = z^2$, $r = z$, $(0, x)$



Intersection with circle and hyperbola

- Hyperbola: $\frac{X^2}{\left(\frac{A}{2K_\phi}\right)^2} - \frac{Y^2}{\left(\frac{1}{2}\right)^2 - \left(\frac{A}{2K_\phi}\right)^2} = 1$
- Circle: $X^2 + \left(Y + \frac{\cot\phi}{2}\right)^2 = \frac{1}{4\sin^2\phi}$



Opt. strat. for opening angle $\pi \geq \varphi_0 \geq \pi/2$!

Intersection: Verification by insertion!

$$X(\phi) = \frac{A}{2} \cdot \frac{\cot\frac{\phi}{2}}{1+\sin\phi} \cdot \sqrt{\left(1+\tan\frac{\phi}{2}\right)^2 - A^2}$$

$$Y(\phi) = \frac{1}{2} \cdot \cot\frac{\phi}{2} \cdot \left(\frac{A^2}{1+\sin\phi} - 1\right)$$

where $A = K_{\phi_0}(\ell_0 - r_0)$