Exercise 4: Expected Lower Bound (4 points)

In the lecture we presented a simple lower bound construction for the exploration of simple grid polygons which results in a competitive ratio of at least $\frac{8}{7}$, the corresponding alternatives are shown in Figure 1. Consider an arbitrary randomized algorithm that shuffles with a fixed probability for any step.

Present a lower bound on the expected value of the competitive ratio if the adversary is aware of the corresponding probabilities.

Figure 1: A simple lower bound construction for simple grid polygons.
Exercise 5:  SmartDFS Example  
(4 points)
Recapitulate the differences between the algorithms DFS, Improved DFS and SmartDFS. What is a split cell, and how is the layer of a cell defined?

Explore the grid graph shown in Figure 2 using the DFS, Improved DFS and SmartDFS algorithms. How many moves are made during the exploration (which includes returning to the start point $s$)? For the first occurrence of a split-cell in the SmartDFS algorithm, draw the polygons, $P_1$, $P_2$ and $Q$ and explain its interpretation!

Figure 2: The grid graph for exploration with various DFS Algorithms.

Exercise 6:  Union and Intersection of gridpolygons  (4 points)
Give a formal proof for the following statement. For any two gridpolygons $P_1$ and $P_2$ we have:

$$E(P_1) + E(P_2) = E(P_1 \cup P_2) + E(P_1 \cap P_2).$$