Repetition: Pledge Algorithm with sensor errors

Pledge-like curve!

**Def.** $\mathcal{K}$ class of curves in $\mathcal{C}_{\text{frei}} \cup \mathcal{C}_{\text{halb}}$, with the following conditions:

1. Parameterized curve with turn-angles and position:
   \[ C(t) = (P(t), \varphi(t)) \text{ mit } P(t) = (X(t), Y(t)) \]
2. Curve surrounds obstacle by Left-Hand-Rule
3. Leaves point is a vertex of an obstacle
4. $\mathcal{C}_{\text{frei}}$-condition holds:
   \[ \forall t_1, t_2 \in C : P(t_1), P(t_2) \in \mathcal{C}_{\text{frei}} \Rightarrow |\varphi(t_1) - \varphi(t_2)| < \pi \]
5. $\mathcal{C}_{\text{halb}}$-condition holds:
   \[ \forall h_i, t \in C : P(t) = P(h_i) \Rightarrow \varphi(t) - \varphi(h_i) < \pi \]
Rep.: Proof correctness

● **Lemma** Curves from $\mathcal{K}$ do not self-intersect.

● **Lemma** Curves from $\mathcal{K}$ hit any edge once.

● **Lemma** For any curve from $\mathcal{K}$: Obstacle will no longer be left, then the curve is enclosed by the obstacle.

● **Theorem** Curves from $\mathcal{K}$ escape, if this is possible.
Rep.: Applications of the model

**Corollary** Compass with deviation maximal $\frac{\pi}{2}$ is sufficient for escaping from a labyrinth.

**Corollary** Axis-parallel scene, hold the direction in the range $\left(-\frac{\pi}{2}, +\frac{\pi}{2}\right)$ and distinguish between horizontal and vertical. Escape!
Rep: Pseudo orthogonal

- Small deviations at the vertices! From global coordinates!
- 1. Condition: Numbers convex vert. = reflex vert. + 4
- Small deviations!
- $\text{div}(e) : e = (v, w)$ smallest deviation from horizontal/vertical line passing durch $v$ und $w$
- $\text{div}(P) := \max_{e \in P} \text{div}(e) \leq \delta$, Def.: $\delta$-pseudo orthogonal scene
Rep: $\delta$-pseudo orthogonal

**Corollary** $\delta$-pseudo-orthogonal scene $P$. Measure angles with precision $\rho$ s.th. $\delta + \rho < \frac{\pi}{4}$. Deviation in the free space always smaller than $\frac{\pi}{4} - 2\delta - \rho$ from global starting direction. Escape from a labyrinth is guaranteed.

1. Distinguish reflex/convex corners: Counting the turns!
2. Max. global deviation of starting direction: Intervall $\pi$
3. Distinguish: Horizontal/Vertical
Rep.: \(\delta\)-pseudo orthogonal scene

- Precision \(\rho\) with \(\delta + \rho < \frac{\pi}{4}\)
- Free-space max. deviation \(\frac{\pi}{4} - 2\delta - \rho\)
- 1. Distinguish reflex/convex corners: Worst-case
Find a target point

- Searching for a given goal: Navigation
- Polygonal environment: Finite number of polygons
- Touch sensor: Hand-Rules
- Start $s$, target $t$, coordinates are given
- Finite storage: I.e. Own coordinates
- BUG Algorithms: Sojourner
Notations

- $|pq|$ distance between $p$ and $q$
- $D := |st|$ distance from start to goal
- $\Pi S$ path of strategy $S$ from start to goal
- $|\Pi S|$ length of the path $\Pi S$
- $\cup \Pi_i$ perimeter of obstacle $P_i$
- Actions:
  1. Move into direction of the target
  2. Follow the wall
- Leave-Points $l_i$, Hit-Points $h_i$
BUG1 Strategie: Lumelsky/Stepanov
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0. \( l_0 := s, \ i := 1 \)

1. From \( l_{i-1} \) move into target direction, until
   (a) Goal is reached: Stop!
   (b) An obstacle is met at \( h_i \).

2. Surround the obstacle \( O \) in cw order — continuously calculate and store the point \( l_i \) on \( O \) closest to \( t \) —, until
   (a) Goal is reached: Stop!
   (b) \( h_i \) is visited again!

3. Move along the shortest path along \( O \) to \( l_i \).

4. Increment \( i \), GOTO 1.
Correctness BUG1 strategy

**Theorem** The strategy BUG1 finds a path from $s$ to $t$, if such a path exists.

**Proof:**

- Sequence of Hit- and Leave-Points $h_i, l_i$
- $|st| \geq |h_1t| \geq |l_1t| \ldots \geq |h_k t| \geq |l_k t|$
Point with smallest distance to $t$: Leave-Point $l_i$

No free movement to $t \Rightarrow$ enclosed

$l_i \neq l_j$, new obstacle!

Finitely many obstacles $\Rightarrow$ correctness
Path length BUG1 Strategy

**Theorem** Let $\Pi_{\text{Bug1}}$ be the path from $s$ to $t$, calculated by the BUG1-strategy. We have: $|\Pi_{\text{Bug1}}| \leq D + \frac{3}{2} \sum_i \text{UP}_i$.

**Proof:**

- Subdivision: Free space path, surrounding
- Surrounding, then shortest path to $l_i$
- $\frac{3}{2} \sum \text{UP}_i$
- Finally: Path $D'$ between the obstacles
**Theorem** \( |\Pi_{\text{Bug1}}| \leq D + \frac{3}{2} \sum_i \text{UP}_i \).

**Proof:** \( D' \) between the obstacles

\[
D' = |sh_1| + |l_1h_2| + \ldots + |l_{k-1}h_k| + |l_k t| \\
\leq |sh_1| + |l_1h_2| + \ldots + |l_{k-1}h_k| + |h_k t| \\
= |sh_1| + |l_1h_2| + \ldots + |l_{k-1}t| \\
\ldots \\
\leq |sh_1| + |l_1t| \leq |sh_1| + |h_1t| = |st| = D
\]
Lower bound?

- Show: Bug1 is $\frac{3}{2}$-competitive
- Surround the obstacles along the path
- **Corollary** Bug1 is $\frac{3}{2}$-competitive
- Adversary strategy for the model
- Actions:
  1. Move into direction
  2. Follow the wall
- Leave-Points $l_i$, Hit-Points $h_i$
Lower bound

**Theorem** For any strategy $S$ (due to the action-model), and for any $K > 0$, there exist a strategy with arbitrary $D > 0$, such that for any $\delta > 0$: $|\Pi_S| \geq K \geq D + \sum UP_i - \delta$.

Arbitrarily large path!
$$|\Pi_S| \geq K \geq D + \sum UP_i - \delta.$$ 

- Virtual horse-shoe, Width $2W$, Thickness $\epsilon \ll \delta$, Length $L$, Distance $D$
- Virtual gets precise: Touch the wall!
- For any strategy $S$
\[ |\Pi_S| \geq K \geq D + \sum \text{UP}_i - \delta. \]

- **Idea:**
  \[ D + W - \sqrt{D^2 + W^2} \leq \delta/2 \text{ and} \]
  \[ L + W - \sqrt{L^2 + W^2} \leq \delta/2, \text{ } L, \text{ } W \text{ large enough!} \]

- \[ |\Pi_S| \geq \sqrt{L^2 + W^2} + L + \sqrt{D^2 + W^2} \]
  \[ \geq D + W + L + W - \delta = D + 2(L + W) - \delta \]

\begin{figure}
\begin{center}
\includegraphics[width=\textwidth]{diagram}
\end{center}
\end{figure}
\[ |\Pi_S| \geq K \geq D + \sum \text{UP}_i - \delta. \]

- Problem: Left and right part! Peri. \(4(L + W)\)
- Inside horse-shoe: \(|\Pi_{I_1}| \geq \sum \frac{1}{2} \text{UP}_i\) non-overlapping
- \(|\Pi_{I_2}| \geq \sum \text{UP}_j\) overlapping, \(r_j\) path back
- Outside horse-shoe: \(|\Pi_A| \geq L + C\) with \(C = \sqrt{D^2 + W^2}\)
- \(L_{A_1} \geq \sum \frac{1}{2} \text{UP}_i\) for non-overlapping
- Altogether: \(|\Pi_S| \geq \sqrt{D^2 + W^2} + \sum \text{UP}_i - 2n\epsilon\)
- \(n \leq \frac{2L}{\delta}, \epsilon \leq \delta^2/(4L)\) gives \(2n\epsilon \leq \delta/2\)
- \(|\Pi_S| \leq D + W + \sum \text{UP}_i - \delta\)
BUG2 strategy

Line $G$ passing $st$, target direction, surround obstacle, shortest curr. distance on $G$, move to target
BUG2 strategy

0. $l_0 := s, j := 1$

1. From $l_{j-1}$ move toward target, until
   (a) Goal is reached: Stop!
   (b) Obstacle is met at $h_j$.

2. Surround obstacle cw order, until
   (a) Goal is reached: Stop!
   (b) Line $G$ passing $st$ is visited at $q$, $|qt| < |h_jt|$ and $qt$ locally free for a move
       $l_j := q, j := j + 1$ and GOTO 1.
   (c) $h_j$ is reached again, no point $q$ of case b) was found.
       Reaching the goal is impossible.
BUG2 strategy: Analysis

- Structural properties
- Correctness and performance
- **Lemma** Bug2 visits finitely many obstacles

Proof, by precondition for the scene!
**Lemma** Let $n_i$ denote the number of intersections between $G$ (line passing $st$) and the obstacle $P_i$. Any boundary point of $P_i$ is visited at most $\frac{n_i}{2}$ time.

- Bug2 defines pairs $(h_j, l_j)$ of hit- and leave points.
- Jumping cond.: $|h_j t| > |l_j t| > |h_{j+1} t|$.
- Any intersection with $P_i$ is only once a leave or a hit point.
- Meet current hit point $\Rightarrow$ Stop.
Bug2 visits boundary points max $\frac{n_i}{2}$ times

- Pairs $(h_j, l_j)$ of hit-leave points
- $\frac{n_i}{2}$ pairs $(h_j, l_j)$
- Only then a surrounding is started
- Point on the boundary only $\frac{n_i}{2}$ times
**Corollary** Bug2 visits the goal, if this is possible.

- Finitely many visits, finitely many surroundings!
- Either goal is found or current hit point is visited again
- Current hit point $\Rightarrow$ no free path from a better point on the boundary. Goal is enclosed!
**Theorem** Let $\Pi_{\text{Bug2}}$ denote the path from $s$ to $t$ designed by BUG2. We have $|\Pi_{\text{Bug2}}| \leq D + \sum_i \frac{n_i \text{UP}_i}{2}$. **Proof:**

- Subdivision: Surroundings, Free path
- $\sum_i \frac{n_i \text{UP}_i}{2}$ follows from the **Lemma**
- Length $D'$ between obstacles

![Diagram of path](attachment:bug2_strategy_performance.png)
BUG2 strategy: Performance

- Length $D'$ between obstacles
- Analogously **BUG1 Theorem** $D' \leq D$
- Altogether:

\[
|\Pi_{\text{Bug2}}| \leq D + \sum_{i} \frac{n_{i} \text{UP}_{i}}{2}.
\]
Compare BUG2 and BUG1

- BUG2 not always better, sometimes worse (Exercise)
- Convex polygons: Optimal
- Many further variants!
- Visibility/Local improvements!
Change I

- Bug1 fully surrounds
- Bug2 avoids, but visits many times
- Change make use of old Leave/Hit Points, **One** order change!
Pseudocode: Change I

0. \( \ell_0 := s, \ i := 1 \)

1. Move from \( \ell_{i-1} \) along line passing \( st \) toward goal, until
   
   (a) Goal is reached: Stop!

   (b) Obstacle is met at \( h_i \).

2. Surround obstacle cw order, until
   
   (a) Goal is reached: Stop!

   (b) Line \( G \) passing \( st \) is visited at \( q, \ |qt| < |h_jt| \) and \( qt \) locally free for a move, \( l_j := q, \ j := j + 1 \) and GOTO 1.
(c) A hit- or leave point $h_j$ or $\ell_j$ with $j < i$ is met. Move back to $h_i$, use ccw order until (a), (b) oder (d) happens.

(d) $h_j$ is reached again, no point $q$ of case b) was found. Reaching the goal is impossible.