

Online Motion Planning MA-INF 1314

Alternative cost measures!

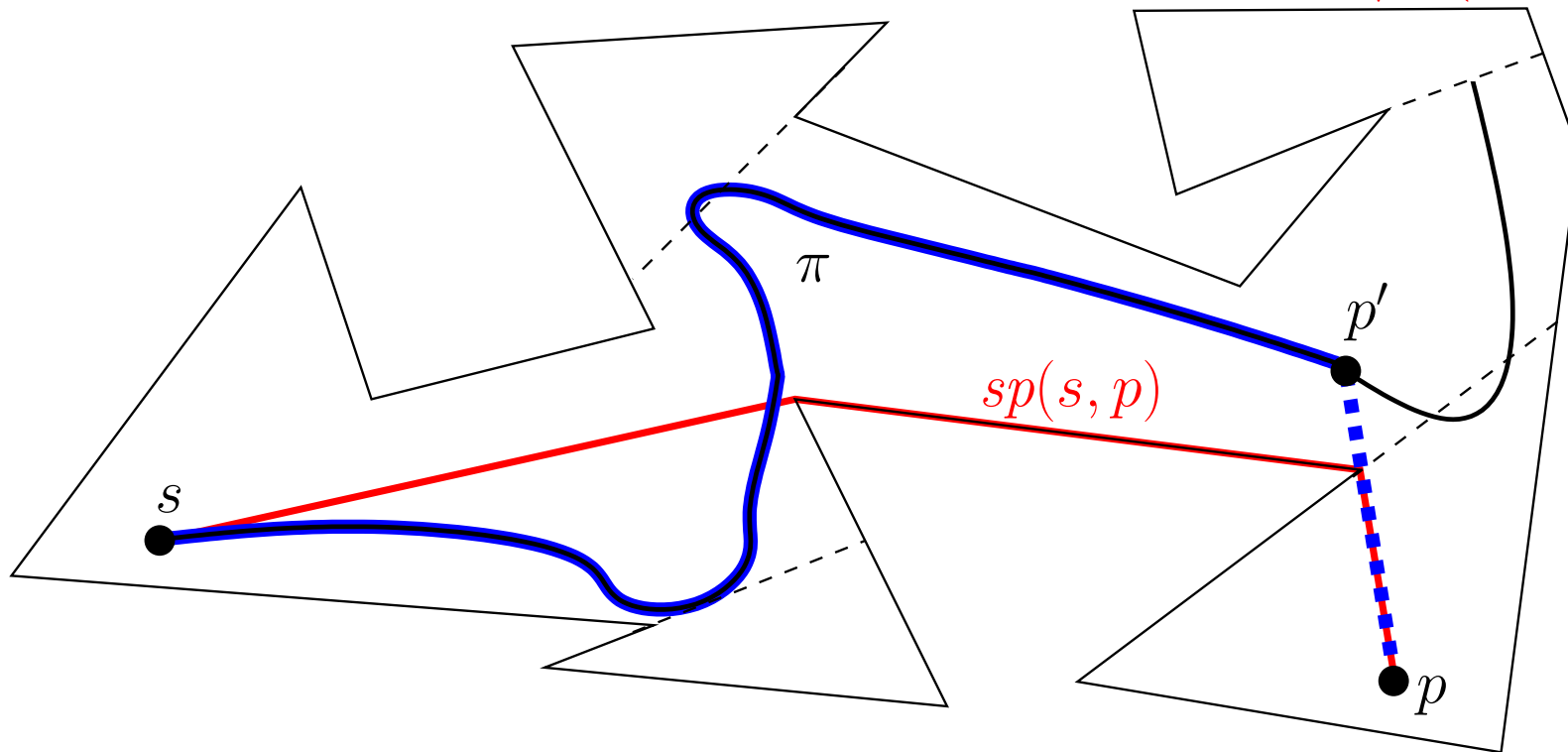
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Rep.: Optimal searchpath

- We have seen:
Searching for a goal (polygon) in general not competitive
- Question: What is a good searchpath (for polygons)?
- Searching: Target point unknown!
- Offline-Searching: Environment is known
- Online-Searching: Environment **un**known

Rep.: Search ratio for polygons

π : Searchpath, quality for π : $SR(\pi, P) = \max_{p \in P} \frac{|\pi_s^{p'}| + |p'p|}{|sp(s, p)|}$ ■ ■



Rep.: Quality measures!

- *Competitive ratio* of search strategy \mathcal{A} in polygons:

$$C := \sup_P \sup_{p \in P} \frac{|\mathcal{A}(s, p)|}{|\text{sp}(s, p)|}$$

Rep.: Search ratio in general

Given: Environment \mathcal{E} , Set of goals $\mathcal{G} \subseteq \mathcal{E}$

Graphs $G = (V, E)$: **Vertices** $\mathcal{G} = V$

Geometric Search $\mathcal{G} = V \cup E$

(Requirement: $\forall p \in \mathcal{E} : |\text{sp}(s, p)| = |\text{sp}(p, s)|$)

Search ratio of a search strategy \mathcal{A} for \mathcal{E} :

$$\text{SR}(\mathcal{A}, \mathcal{E}) := \sup_{p \in \mathcal{G}} \frac{|\mathcal{A}(s, p)|}{|\text{sp}(s, p)|} \blacksquare$$

Optimal search ratio:

$$\text{SR}_{\text{OPT}}(\mathcal{E}) := \inf_{\mathcal{A}} \text{SR}(\mathcal{A}, \mathcal{E}) \blacksquare$$

Rep.: Search ratio approximation

- *Competitive ratio* : $C := \sup_{\mathcal{E}} \sup_{p \in \mathcal{G}} \frac{|\mathcal{A}(s, p)|}{|\text{sp}(s, p)|}$
- *Search ratio*: $\text{SR}(\mathcal{A}, \mathcal{E}) := \sup_{p \in \mathcal{G}} \frac{|\mathcal{A}(s, p)|}{|\text{sp}(s, p)|}$
- *Optimal search ratio*: $\text{SR}_{\text{OPT}}(\mathcal{E}) := \inf_{\mathcal{A}} \text{SR}(\mathcal{A}, \mathcal{E})$
- *Approximation: \mathcal{A} search-competitiv*

$$C_s := \sup_{\mathcal{E}} \frac{\text{SR}(\mathcal{A}, \mathcal{E})}{\text{SR}_{\text{OPT}}(\mathcal{E})}$$

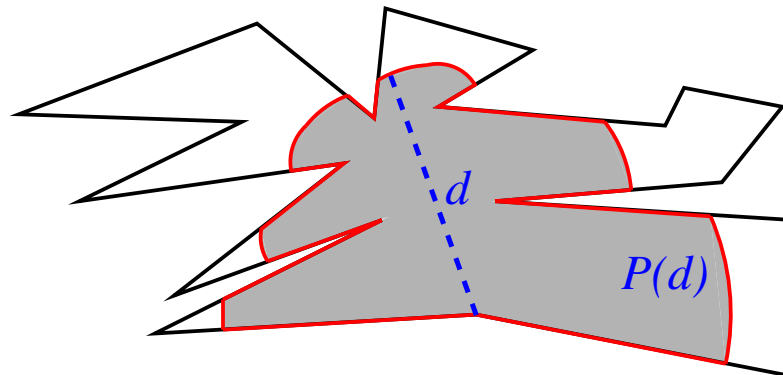
- Comparison not against SP, but against best possible SR

Rep.: Depth-restricted exploration

Def. Exploration-Strategy Expl for \mathcal{E} is called **depth-restrictable**, if we can derive a strategy $\text{Expl}(d)$ such that:

- $\text{Expl}(d)$ explores \mathcal{E} up to depth $d \geq 1$
- return to s after the exploration
- $\text{Expl}(d)$ is C -competitiv, i.e., $\exists C \geq 1 : \forall \mathcal{E}$:

$$|\text{Expl}(d)| \leq C \cdot |\text{Expl}_{\text{OPT}}(d)|.$$



Rep.: Searchpath approximation

Algorithm

- Explore \mathcal{E} by increasing depth: $\text{Expl}(2^i)$ für $i = 1, 2, \dots$

Lemma:

- Roboter without vision system
- Environment \mathcal{E}
- Expl_{ONL} : C -competitive, depth-restrictable, online exploration strategy for \mathcal{E}
(d. h. $|\text{Expl}(d)| \leq C \cdot |\text{Expl}_{\text{OPT}}(d)|$)

\Rightarrow Algorithm gives $4C$ -Approximation of optimal search path!

Rep.: Searchpath approximation proof

$$|\Pi_{\text{Expl}_{\text{opt}}}(d)| \leq d \cdot (\text{SR}(\Pi_{\text{opt}}) + 1) \quad (1)$$

$$\begin{aligned} \text{SR}(\Pi) &\leq \frac{\sum_{i=1}^{j+1} |\Pi_{\text{Expl}}(2^i)|}{2^j + \varepsilon} \\ &\stackrel{(\text{Ass.})}{\leq} \frac{C}{2^j} \sum_{i=1}^{j+1} |\Pi_{\text{Expl}_{\text{opt}}}(2^i)| \stackrel{(1)}{\leq} \frac{C}{2^j} \sum_{i=1}^{j+1} 2^i \cdot (\text{SR}(\Pi_{\text{opt}}) + 1) \\ &\leq C \cdot \left(\frac{2^{j+2} - 1}{2^j} \right) \cdot (\text{SR}(\Pi_{\text{opt}}) + 1) \leq 4C \cdot (\text{SR}(\Pi_{\text{opt}}) + 1) \end{aligned}$$

Rep.: Applications

⇒ Searchpath approximation of factor 4

- Graphs: Online and Offline!

CFS ($C = 4 + \frac{8}{\alpha}$) depth-restrictable!!

- But: Factor depends on rope length $(1 + \alpha)r$ by depth r
- CFS sometimes explores more than d (precisely $(1 + \alpha)d$)

⇒ $\text{Expl}(d)$ not comparable to $\text{Expl}_{\text{OPT}}(d)$ ■

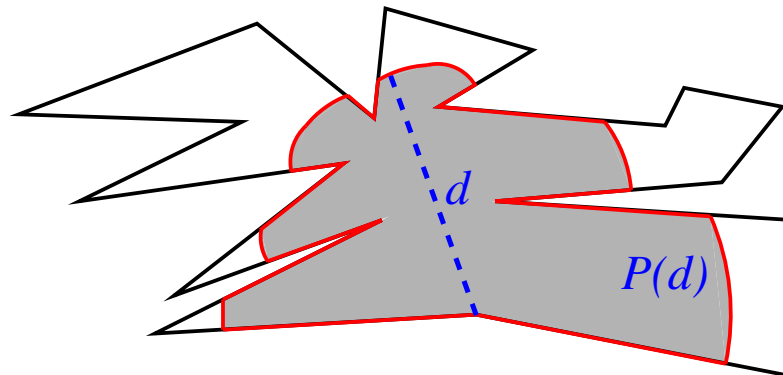
- Workaround: Compare $\text{Expl}(d)$ with $\text{Expl}_{\text{OPT}}(\beta \cdot d)$

β -depth restricted exploration

Def. Exploration strategy Expl for \mathcal{E} is denoted as β -depth restrictable, if we can derive a strategy $\text{Expl}(d)$ such that:

- $\text{Expl}(d)$ explores \mathcal{E} only up to depth $d \geq 1$
- returns to the start s
- $\text{Expl}(d)$ is C_β -competitiv, i.e., $\exists C_\beta \geq 1, \beta > 0 : \forall \mathcal{E}$:

$$|\text{Expl}(d)| \leq C_\beta \cdot |\text{Expl}_{\text{OPT}}(\beta \cdot d)|.$$



Searchpath approximation

Theorem:

- Agent without vision
 - Environment \mathcal{E}
 - Expl: C_β -competitive, β -depth restrictable, online exploration strategy for \mathcal{E} , (i.e., $|\text{Expl}(d)| \leq C_\beta \cdot |\text{Expl}_{\text{OPT}}(\beta \cdot d)|$)
- \Rightarrow Algorithm (exploration/double depth) gives a $4\beta C_\beta$ -approximation of the optimal searchpath ■

Corollary: Unknown graphs, Algorithm with CFS is $4(1 + \alpha)(4 + \frac{8}{\alpha})$ -approximation of optimal searchpath ■

Searchpath Approximation Proof

$$|\Pi_{\text{Expl}_{\text{opt}}(d)}| \leq d \cdot (\text{SR}(\Pi_{\text{opt}}) + 1) \quad (2)$$

$$\begin{aligned} \text{SR}(\Pi) &\leq \frac{\sum_{i=1}^{j+1} |\Pi_{\text{Expl}(2^i)}|}{2^j + \varepsilon} \\ &\leq \frac{C_\beta}{2^j} \sum_{i=1}^{j+1} |\Pi_{\text{Expl}_{\text{opt}}(\beta 2^i)}| \stackrel{(2)}{\leq} \frac{C_\beta}{2^j} \sum_{i=1}^{j+1} \beta 2^i \cdot (\text{SR}(\Pi_{\text{opt}}) + 1) \\ &\leq C_\beta \beta \cdot \left(\frac{2^{j+2} - 1}{2^j} \right) \cdot (\text{SR}(\Pi_{\text{opt}}) + 1) \leq 4\beta C_\beta \cdot (\text{SR}(\Pi_{\text{opt}}) + 1) \end{aligned}$$

Non approximation results: Theorem

No constant approximation of the search ratio!

1. Planar graph $G = (V, E)$ multiple edges, goal set V .
2. General graph $G = (V, E)$ goal set V .
3. Directed graph $G = (V, E)$ goal set E and V .



Counter examples, lower bound! Blackboard! 

Searching with vision!

Problem: Return path from $\text{last}(d)$ to s has length $\leq d$, might be false! ■ But: $\text{sp}(\text{last}(d), s) \leq |\pi_{\text{OPT}_s}^{\text{last}(d)}|$ ■

Robot still has to move to t ■

Theorem:

- Roboter **with** vision
 - Environment \mathcal{E}
 - Expl: C_β -competitive, β -depth restrictable, Online Explorationstrategy for \mathcal{E}
(i.e. $|\text{Expl}(d)| \leq C_\beta \cdot |\text{Expl}_{\text{OPT}}(\beta \cdot d)|$)
- ⇒ Algorithm gives **$8\beta C_\beta$** -Approximation of optimal search ratio. ■

Proof of the Theorem

$$\text{SR}(\Pi_{\text{opt}}) \geq \frac{|\pi_{\text{OPT}s}^{\text{last}(d)}|}{d} \geq \frac{|\Pi_{\text{Expl}_{\text{opt}}(d)}|}{2d} \Leftrightarrow |\Pi_{\text{Expl}_{\text{opt}}(d)}| \leq 2d \cdot \text{SR}(\Pi_{\text{opt}})$$

Ratio against search path:

$$\begin{aligned} \frac{\sum_{i=1}^{j+1} |\Pi_{\text{Expl}_{\text{onl}}(2^i)}|}{2^j} &\leq C_\beta \cdot \frac{\sum_{i=1}^{j+1} |\Pi_{\text{Expl}_{\text{opt}}(\beta 2^i)}|}{2^j} \leq 2C_\beta \cdot \frac{\sum_{i=1}^{j+1} \beta 2^i \text{SR}(\Pi_{\text{opt}})}{2^j} \\ &\leq 8\beta C_\beta \cdot \text{SR}(\Pi_{\text{opt}}) . \end{aligned}$$

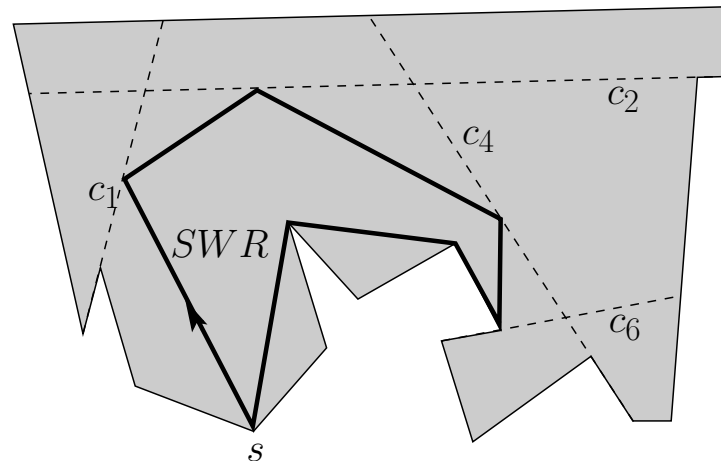
Outlook: Applications!

- Simple polygon, Offline: SWR ($C_\beta = 1 = \beta$)
⇒ 8-Approximation■
- Rectilinear Polygons, Online: Greedy-Online ($C_\beta = \sqrt{2}, \beta = 1$)
⇒ $8\sqrt{2}$ -Approximation■
- Simple Polygons, Online: PolyExplore ($C_\beta = 26, \beta = 1$)
⇒ 212-Approximation■

Exploration!■

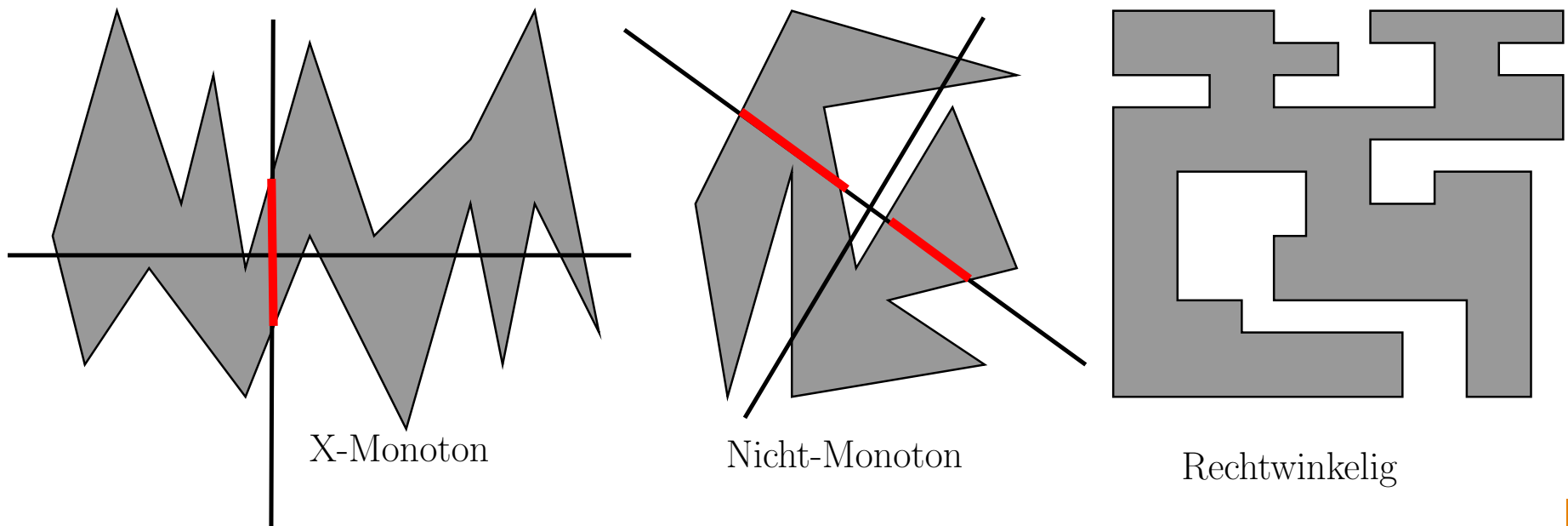
Simple Polygon **Offline**

- Optimal exploration tour■
- Agent with vision, start point s , boundary■
- Polygon is fully known■
- Depth restriction■
- First: General approach. Then: Depth restriction!■



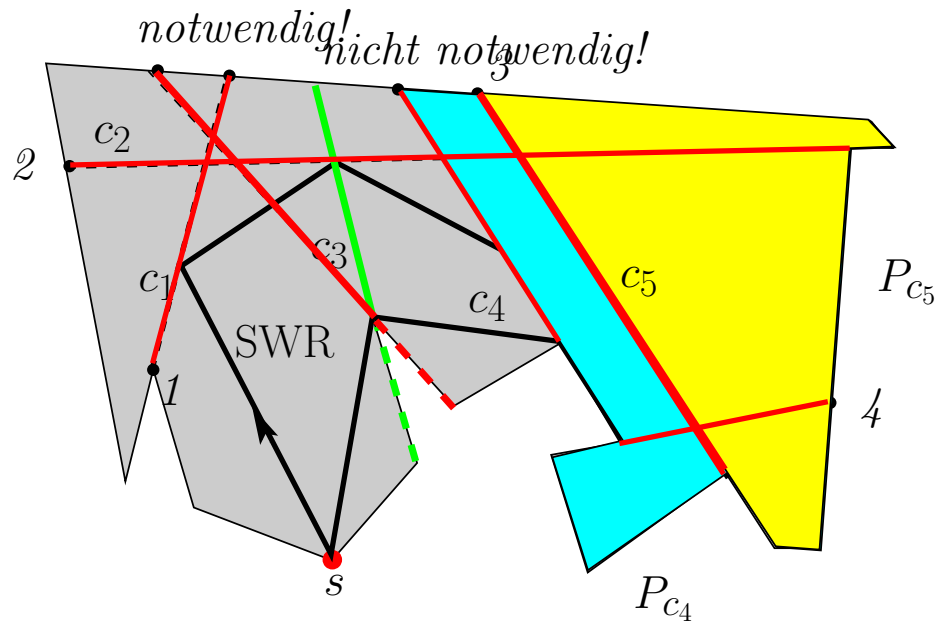
Simple polygon SWR **Offline**

- Standard approach (a bit simpler) ■
- Monotone polygons: Monoton w.r.t l ■
- Rectilinear polygons ■
- Rectilinear and monotone polygons: SWR in $O(n)$ ■



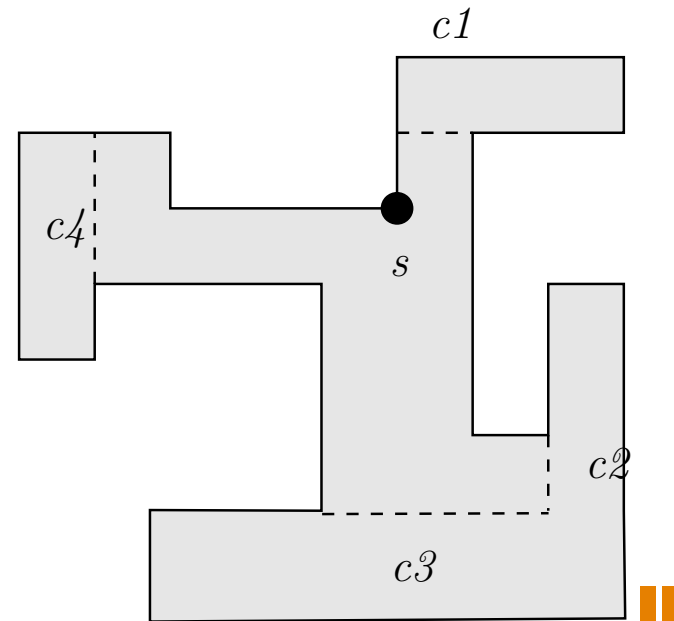
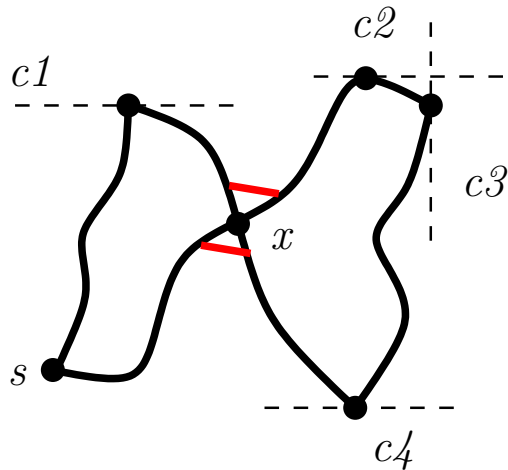
In general: Important segments? Def.

- a) (Cuts) Extension of reflex vertex
- b) Necessary cuts (w.r.t. s)
- c) Dominance-Relationship $P_{c_i} \subseteq P_{c_j}$
- d) Essential cuts
- e) Order of the essential cuts

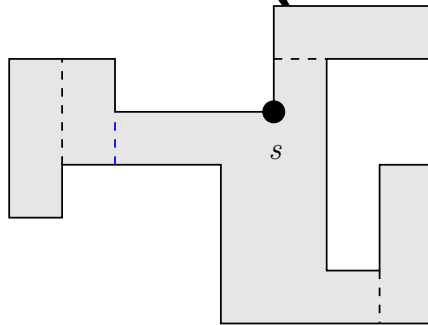


Ordnung along the boundary Lem.

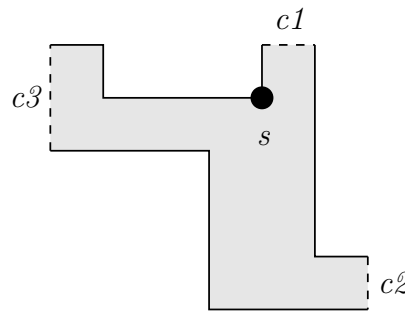
- Rectilinear polygon
- Essential cuts intersect at most once
- SWR visits cuts by order around boundary
- Contradiction! Shortcut!
- $O(n)$ Algorithm!!



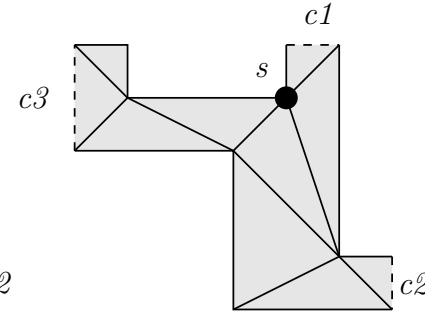
SWR (RW Polygon) $O(n)$ Theo.



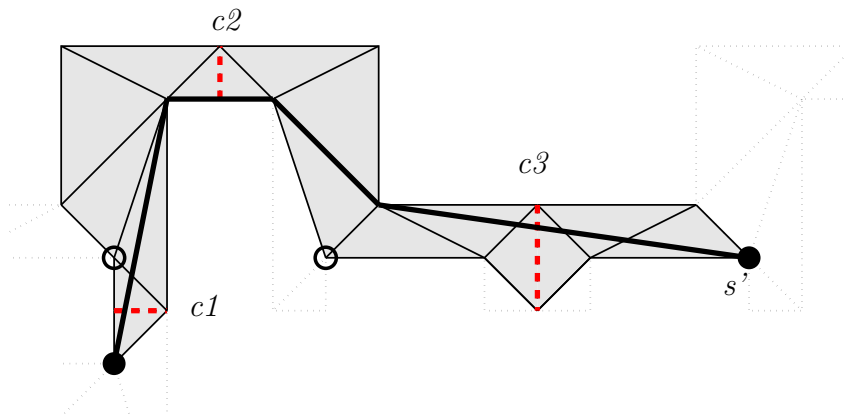
(i) Wesentliche Cuts
 $O(n)$



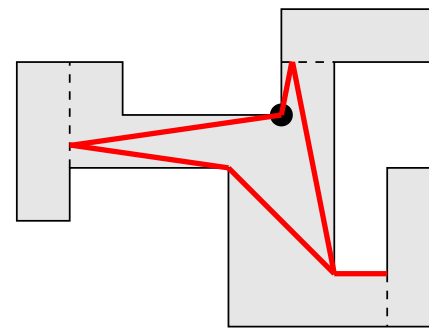
(ii) Abschneiden!
 $O(n)$



(iii) Triangulation
 $O(n)$



(iv) Spiegeln und Ausrollen!!
(v) Weg berechnen
 $O(n)$



(vi) Zurckklappen!
 $O(n)$

SWR (Rect. polygon) depth restriction?

- Ignore cuts with distance $> d$, Shortest path to cut
- Ignore a cut here, Algorithm as before
- $\text{Expl}(d) = \text{Expl}_{\text{OPT}}(d)$
- **Theorem:** 8 Approximation of optimal search path!

