Online Motion Planning MA-INF 1314
Searching

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Rep.: Navigation

- Touch sensor, Target coordinates, Start $s$, Target $t$, Storage, Sojourner
- Actiony: 
  - Move toward the target
  - Move along the boundary
  - Sequence of Leave-Points $l_i$, Hit-Points $h_i$
Rep.: BUG1 Strategy: Lumelsky/Stepanov

Toward target, surround obstacle, best leave point, toward target!
Rep: Analysis BUG1 Strategy

- **Theorem** Strategy BUG1 is correct!
- **Theorem** Successful Big1-path $\Pi_{\text{Bug1}}$ from start $s$ to target $t$:
  $$|\Pi_{\text{Bug1}}| \leq D + \frac{3}{2} \sum_i UP_i.$$
- **Theorem** For any strategy $S$, for arbitrary large $K > 0$, there exists examples for any $D > 0$, such that for any arbitrarily small $\delta > 0$ we have:  
  $$|\Pi_S| \geq K \geq D + \sum UP_i - \delta.$$
- **Korollar** Bug1 is $\frac{3}{2}$-competitive against any online strategy.
Rep. LB: \(|\Pi_S| \geq K \geq D + \sum UP_i - \delta\)

- Virtual horse-shoe, width \(2W\), thickness \(\epsilon \ll \delta\), length \(L\), dist. \(D\)
- Virtual gets concrete by touch
- Roughly surround any obstacle, by any strategy!
Rep.: BUG2 Strategy

Line $G$ passing $st$, toward target, surround obstacle, shorter distance on $G$, toward target!
• **Lemma** Let $n_i$ denote the number of intersection of $G$ with relevant obstacle $P_i$. Bug2 meets any point on $P_i$ at most $\frac{n_i}{2}$ times.

• **Corollary** Bug2 is correct!

• **Theorem** Bug2-path $\Pi_{\text{Bug2}}$ from $s$ to $t$. We have:

$$|\Pi_{\text{Bug2}}| \leq D + \sum_i \frac{n_i \text{UP}_i}{2}.$$
Rep.: Change I

Change I, use former Leave/Hit Points once for !

Theorem: Change I requires at most path length $|\Pi_{\text{Change I}}| \leq D + 2 \sum_i UP_i$. This is a tight bound!

Exercise!
Different models

- Sensor with range: Circle around current point
- Short-cut for BUG2: VisBug
- Many others
Searching for a goal!

- Coordinates of the target unknown: Searching vs. Navigation
- Polygonal environment
- Full sight: Visibility polygon
- **Def.** Let $P$ be a simple polygon and $r$ a point with $s \in P$. The visibility polygon of $r$ w.r.t. $P$, $\text{Vis}_P(r)$, is the set of all points $q \in P$, such that the segment $\overline{rq}$ is fully inside $P$.
- Alg. Geom.: Compute in $O(n)$ time! Offline!
Corridors (without sight)

- 2-ray search: Find door along a wall!
- Compare to shortest path to the door, competitive?
- Reasonable strategy: Depth $x_1$ right, depth $x_2$ left and so on
- Start-situation: $2x_1 \geq C\epsilon$, for any $C > 0$ ex. $\epsilon$
- Additive constant or goal is at least step 1 away!
- Local worst-case, not visited at $d$, once back!
- Find strategy, such that: $\sum_{i=1}^{k+1} 2x_i + x_k \leq Cx_k$
Corridors

- Worst-case, not visited at \( d \), once back!
- Find strategy, such that:
  \[
  \sum_{i=1}^{k+1} 2x_i + x_k \leq Cx_k
  \]
- Minimize:
  \[
  \frac{\sum_{i=1}^{k+1} 2x_i + x_k}{x_k} = 1 + 2\frac{\sum_{i=1}^{k+1} x_i}{x_k}
  \]
- \( x_i = 2^{i-1} \), gives ratio \( C = 9 \)
- Proof: Blackboard!

\[
\begin{align*}
32 & \rightarrow 8 \rightarrow 2^{\frac{1}{2}} \rightarrow 4 \rightarrow 16
\end{align*}
\]
Theorem Opt. of exponential solution: Gal 1980

- **Strategy:** Sequence \( X = f_1, f_2, \ldots \)

- **Minimize functional** \( F_k(f_1, f_2, \ldots) := \frac{\sum_{i=1}^{k+1} f_i}{f_k} \) for alle \( k \)

- More precisely \( \inf_Y \sup_k F_k(Y) = C \) und \( \sup_k F_k(X) = C \)

- In general: Functional \( F_k \) continuous/unimodal: Unimodal:
  \( F_k(A \cdot X) = F_k(X) \) and \( F_k(X + Y) \leq \max\{F_k(X), F_k(Y)\} \)

- Some other helpful conditions!

- I.e.: \( F_{k+1}(f_1, \ldots, f_{k+1}) \geq F_k(f_2, \ldots, f_{k+1}) \)

- **Theorem** Exponential function minimizes \( F_k \):

  \[
  \sup_k F_k(X) \geq \inf_a \sup_k F_k(A_a)
  \]

  mit \( A_a = a^0, a^1, a^2, \ldots \) und \( a > 0 \).
Example: Exponential function

- $F_k(f_1, f_2, \ldots) := \frac{\sum_{i=1}^{k+1} f_i}{f_k}$ for all $k$.
- Unimodal $F_k(A \cdot X) = F_k(X)$ and $F_k(X + Y) \leq \max\{F_k(X), F_k(Y)\}$.
- $\frac{\sum_{i=1}^{k+1} A \cdot f_i}{A \cdot f_k} = \frac{\sum_{i=1}^{k+1} f_i}{f_k}$
- $F_k(X + Y) \leq \max\{F_k(X), F_k(Y)\}$.
- Follows from $\frac{a}{b} \geq \frac{c}{d} \iff \frac{a+c}{d+b} \leq \frac{a}{b}$
- Simple equivalence!
- Optimize: $f_k(a) := \sum_{i=1}^{k+1} a^i$
- Minimized by $a = 2$
Theorem Gal 1980

If functionally $F_k$ has the following properties:

i) $F_k$ is continuous,

ii) $F_k$ is unimodal: $F_k(A \cdot X) = F_k(X)$ and $F_k(X + Y) \leq \max\{F_k(X), F_k(Y)\}$,

iii) $\liminf_{a \to \infty} F_k\left(\frac{1}{a^{k+i}}, \frac{1}{a^{k+i-1}}, \ldots, \frac{1}{a}, 1\right) = \liminf_{\epsilon_{k+i}, \epsilon_{k+i-1}, \ldots, \epsilon_1 \to 0} F_k\left(\epsilon_{k+i}, \epsilon_{k+i-1}, \ldots, \epsilon_1, 1\right)$,

iv) $\liminf_{a \to 0} F_k\left(1, a, a^2, \ldots, a^{k+i}\right) = \liminf_{\epsilon_{k+i}, \epsilon_{k+i-1}, \ldots, \epsilon_1 \to 0} F_k\left(1, \epsilon_1, \epsilon_2, \ldots, \epsilon_{k+i}\right)$,

v) $F_{k+1}(f_1, \ldots, f_{k+i+1}) \geq F_k(f_2, \ldots, f_{k+i+1})$.

Then: $\sup_k F_k(X) \geq \inf_a \sup_k F_k(A_a)$ with $A_a = a^0, a^1, a^2, \ldots$ and $a > 0$. 
Application m-ray search

- Arbitrary $m$, not competitive, Fig. 1
- $2m - 1$ vs. $1$
- Fixed $m$, infinite rays
- Ass.: Rays in fixed order and increasing depth
- Tupel $(f_j, J_j)$: depth, next visit
Anwendung m-Wege Suche

- **Ass.**: \((f_j, J_j), J_j = j + m, f_j \geq f_{j-1}\)
- Visit rays in fixed order, increasing depth
- \(F_k(f_1, f_2, \ldots) := \frac{f_k + 2 \sum_{i=1}^{k+m-1} f_i}{f_k}\) für alle \(k\).
- (Gal) Exp.-function minimizes \(F_k\): \(\sup_k F_k(X) \geq \inf_a \sup_k F_k(A_a)\)
  with \(A_a = a^0, a^1, a^2, \ldots\) and \(a > 1\), optimal \(a = \frac{m}{m-1}\)
- Ratio: \(C = 1 + 2m \left(\frac{m}{m-1}\right)^{m-1}\) opt.
m-ray search

- **Lemma** There is an optimal m-ray search strategy \((f_1, f_2, \ldots)\) that visits the rays in a fixed order and with increasing depth.
- periodic and monotone: \((f_j, J_j), J_j = j + m, f_j \geq f_{j-1}\)
- Second part: Proof blackboard! Change strategy! Conditions!
Other approach: Optimality for equations!

- Reasonable strategy, ratio: \[ \frac{\sum_{i=1}^{k+1} 2x_i + x_k}{x_k} = 1 + 2\sum_{i=1}^{k+1} x_i \]
- Ass.: \( C \) optimal, \[ \frac{\sum_{i=1}^{k+1} x_i}{x_k} \leq \frac{(C-1)}{2} \]
- There is strategy \((x'_1, x'_2, x'_3 \ldots)\) s. th. \[ \frac{\sum_{i=1}^{k+1} x'_i}{x'_k} = \frac{(C-1)}{2} \] for all \( k \)
- Monotonically increasing in \( x'_j \) \((j \neq k)\), decreasing in \( x'_k \)
- First \( k \) with: \[ \frac{\sum_{i=1}^{k+1} x_i}{x_k} < \frac{(C-1)}{2} \], decrease \( x_k \)
- \[ \frac{\sum_{i=1}^{k} x_i}{x_{k-1}} < \frac{(C-1)}{2} \]!, \( x_{k-1} \) decrease etc., monotonically decreasing sequence, bounded, converges! Non-constructive!
Other approach: Optimality for equations!

- Set: \[ \sum_{i=1}^{k+1} \frac{x_i'}{x_k'} = \frac{(C-1)}{2} \text{ for all } k \]
- \[ \sum_{i=1}^{k+1} x_i' - \sum_{i=1}^{k} x_i' = \frac{(C-1)}{2} (x_k' - x_{k-1}') \]
- Thus: \[ C' \left( x_k' - x_{k-1}' \right) = x_{k+1}', \text{ Recurrence!} \]
- Solve a recurrence! Analytically! Blackboard!
- Characteristical polynom: No solution \( C' < 4 \)
- \[ x_i' = (i + 1)2^i \text{ with } C' = 4 \text{ is a solution! Blackboard! Optimal!} \]