Online Motion Planning MA-INF 1314

Sample questions for the oral exam

Remarks: This is a walkthrough of the lecture in the summer semester 2016.
Note that, in the oral exam we could also make use of precise example instances.
For example: Please apply and explain the strategy (or algorithm) XY for this example.
Exemplify the analysis. At some points we might go into the depth
in order to justify very good marks for the exam. The list of questions is given
without any claim of completeness. It is not sufficient to give One-Sentence answers,
we normally expect justifications of your statements.
Of course we expect that the competitive ratios and/or the computation bounds
(lower and upper bounds) of the corresponding problems can be presented.

Chapter 1: Labyrinths, grids and graphs

1.1-1.3: Shannon, general graphs, lower bound, competitive ratio

+   Define the notion of the *competitive ratio*.
         Give an example where the additive constant is required.
         Give an example where the additive constant *has* to be omitted.
+   Upper bound: Present a problem example and a strategy with a constant
         competitive ratio.
         Lower bound: Present a problem example where no competitive strategy exists.
+   Describe the configuration of *Shannons Mouse Strategy.*
         Prove the correctness of the strategy.
+   What is the difference between the *exploration of a graph* and
         searching for a goal inside a graph w.r.t. a competitive ratio?
+   Assume that we would like to explore all edges and all vertices
         of a graph (online) and return to the start!
         Give a competitive strategy and present the competitive ratio.
         Give a lower bound for the ratio for any strategy, and explain the proof.
         Present the optimal strategy.
+   Present the lower bound construction of the above online scenario, if
         the agent need not return to the start.
+   Discuss the competitive efficiency of DFS and BFS in graphs.
+   What can we say about the offline version of the above exploration tasks?
         Is it possible to compute the optimal exploration path for a graph?
         What is known about the computational complexity?
+   Assume that we only would like to explore the vertices of an unknown graph?
         Does a competitive online strategy exist? Justify your answer.

1.4: Grid environment

+   Explain the exploration task for simple grid polygons as discussed in the lecture.
+   Why can we no longer apply the lower bound construction for general graphs?
+   Present the lower bound construction for the competitive ratio. Sketch the proof of the lower bound. Why is it important to have conjunctions between the different blocks?
+   Present the task and the idea of SmartDFS. Explain the improvements against pure DFS.
+   Apply the SmartDFS strategy at the following example. Explain the efficiency formula of SmartDFS. Is this a tight bound for SmartDFS? In what sense is SmartDFS optimal?
+   Sketch the proof of the analysis (C+1/2E-3) of SmartDFS: Explain the corresponding statements (i.e.: Offset-Lemma, Excess-Lemma, Shortest-Path-Lemma, etc.) and put them together for the full proof. Explain the decomposition idea in the following example!
+   Give a formal proof for the Excess-Lemma.
+   Give a formal proof for the Shortest-Path-Lemma.
+   Present the competitive ratio of SmartDFS. Is this ratio tight for SmartDFS?
+   Sketch the proof of the competitive ratio of SmartDFS: Explain the corresponding statements (i.e.: Narrow-passages-Lemma, Edge-Lemma, Exploration-Lemma, etc.) and present the inductive proof.
+   Explain the idea of the exploration improvement for SmartDFS for grid-environments without narrow passages.
+   What is the idea of the online Spanning-Tree-Algorithm for the exploration of *general* grid polygons. What is the precise model? Is this model actually different from the model for SmartDFS?
+   Present the spanning tree algorithm for the following example and sketch the analysis for the number of steps.
+   Is the bound for the STC-Algorithms tight? What does the bound mean for the competitive ratio of the STC-Algorithms?
+   What is the competitive ratio of the spanning tree algorithms?

1.4: Constrained graph exploration

+   What kind of tasks do we consider for the constrained-graph-exploration case?
+   Explain the different models!
+   How do we derive the Accumulator-Variant from the Rope-Variant? The running time is extended by a factor. Present and explain this factor.
+   How can we use the Accumulator-Variant for the construction of an offline algorithm. Why are such approximation helpful?

+ What is the problem of using a pure DFS strategy?
+ Explain the principle of the CFS-Algorithm
  (i.e.: DFS, bDFS, Pruning, Merge, List of trees, etc.).
  What kind of invariants do we require? (Lemma 1.23)
+ Apply the CFS-algorithm to the following example,
  explain its structure and the running time.
+ Analyse the running time of the CFS Algorithm:
  > a) by total edge visits (present the 3 different costs)
  > b) subdivided into edge and vertex visits
+ Proof the invariants of Lemma 1.23.
+ What kind of changes have to be done, if the depth of the
  graph is not known from the very beginning?
  Where does the analysis change now?
  What is the effect on the running time?
+ Present the invariants of this CFS variant in
  this case (Lemma 1.27).
+ Explain the model for the mapping-of-a-graph problem.
  What is meant by the local cyclic order?
  Why is it clear that at least one marker is required in the given model?
+ Explain the Marker-Algorithm for the mapping of a graph.
  Apply the Marker-Algorithm in the following example.
  Analyse the running time.
  What kind of costs can be considered?

Chapter 2: Polygonal environments

2.1: Escape from the labyrinth

+ Explain the model and the idea of the pledge-algorithm.
+ Proof the correctness of the pledge-algorithm.
  Recapitulate the essential statements (i.e.: (in case of fail) structure of
  last part of the path, intersection-free, cw/ccw-loops and
  the consequences) and put them together for the proof.
+ Exemplify the pledge algorithm at the following example.
+ What kind of sensor errors might disturb the correctness of the pledge?
+ Define the formal condition for a curve C, that allows an agent to
  escape from a labyrinth, if the agent orient itself on C.
+ What kind of drawbacks do the C-Half and C-Free condition resolve?
  Present examples!
+ Proof the correctness of the class of curves K.
+ Show that a curve from class K is intersection free.
+ Assume that the total number of segments of the labyrinth is a priori known.
  After how many edge visits do we know, that the pledge cannot escape?
  Explain your answer!
+ Why does an erroneous compass of total error at most pi/2 be sufficient

for escaping from the labyrinth?
+ What does the notion of a delta-pseudo-orthogonal polygon mean?
What is the idea for the navigation by pledge in such environments?

2.2: Online Navigation with touch sensor and known coordinates

+ Explain the model for the Rover-Bug and Sojourner configurations.
+ Apply and explain the strategies Bug1, Bug2, Change1 and Change2.
Analyse the overall path length, respectively.
What is the corresponding intention of the different strategies.
+ Give examples for the tightness of the strategies Bug1 and Bug2.
+ Present the ideas for the correctness of the above strategies.
+ Present the general lower bound construction for strategies in
this model.

Chapter 3: Online searching for objects

3.1: 2-ray search an the Theorem of Gal

+ Formalise the 2-ray search problem and show that it is
a problem configuration for the Theorem of Gal.
+ Summarise the conditions for the Theorem of Gal.
What is the main condition in Theorem of Gal?
+ How did we solve 2-ray search problem by the application
of the Theorem, explain the computational details?
+ Is the above solution unique?
+ Present an arbitrary functional of your own choice
that fulfils the conditions of the Theorem of Gal.
Explain and prove its uni-modality.
+ Consider the given functional: What does uni-modality mean?
Proof the uni-modality for the given functional.
+ Design a functional for the following problem example
(example will be presented)!
+ How can we generalise the above situation to m-rays?
+ Proof that for the m-ray search problem there is always
a monotone and periodic optimal competitive strategy.
+ Why is it not possible to search for a point in a polygon
with a constant competitive ratio? Present the lower bound
construction!
+ What does the paradigm "optimality by equality" mean?
Give an example for the application of this paradigm
w.r.t. 2-ray search.
+ How did we find the best strategy for 2-rays with bounded depth?
Explain the general idea.

3.2: Searching for a ray in the plane

+   Describe the general configuration for searching for a ray in the plane.
+   Present a lower bound on the competitive ratio:
           a) Special rays
           b) Discretisation
           c) Application of the Theorem of Gal
+   Present the upper bound achieved by the best Logarithmic Spiral.
           a) What are the features of Logarithmic Spiral (Definition, Length, etc.)?
           b) What is the worst case situation for the ratio?
           c) For what point do we optimise the spiral?
           d) What can the adversary do afterwards?
           e) Why does this procedure give the best spiral?
+   Explain the situation of the Window-Shopper-Problem.
+   Explain, how we find the optimal strategy:
           a) Formalise the conditions for the strategy
           b) How do we resolve the parameters?
           c) Proof of optimality

3.2: Searching in a street polygon

+   Can solve the problem "searching-for-a-goal-in-a-polygon"
with a competitive strategy?
+   Give the formal definition of a street-polygon.
Present the structural property w.r.t. the reflex vertices.
+   Explain the structural properties at the following example.
+   Present a lower bound on the competitive ratio!
+   What is a funnel polygon?
Why is it sufficient to find a competitive strategy for the funnel situation?
+   Sketch the ideas for the design of the optimal strategy for opening
           angle varphi>=pi/2.
I) What is the general lower bound?
II) What are the conditions for a subpath of length w
        between two opening angles varphi_1 and varphi_2?
           a) If the corresponding reflex vertices do not change.
           b) If the corr. reflex vertices change.
III) What idea do we apply for the design of the strategy?
IV) How did we construct and analyse the resulting curve?
    (Explain the intersection of parabola and circle)
+   Sketch the idea for the design of the optimal strategy for opening
           angle varphi<=pi/2.
What is the main difference to the above case?
+   We can design the strategy for angles varphi<=pi/2 (varphi>=pi/2)
also for larger (smaller) opening angles. The curve is well-defined.
What is the problem with the resulting curve?
+   Exemplify the strategy in the following example.

3.4:   Optimal search paths

+   Explain the concept of the optimal search path and the
    search ratio for the case of simple polygons.
    Why did we introduce this concept?
+   Present an example of an optimal search path for
    a given environment.
+   Present an example for the online approximation of
    the optimal search path in graphs.
+   What is the precise relationship between depth-restriced
    competitive online exploration and the online approximation of
    the optimal search path and search ratio?
+   Present the general proofs for the online approximation of the
    optimal search ratio by the doubling heuristic:
         a) Explain the algorithmic idea
         b) Present the requirements
         c1) Prove the 4C approximation in the case without vision
         c2) Prove the 8C approximation in the case with vision
+   Explain the introduction of the stretch factor \beta and the
    stretch ratio C_\beta. How does the factors influence the general framework.
+   Present examples for the application of the general approximation
    framework.
+   Give an example that shows that there is not always a constant competitive
    approximation of the optimal search ratio for graphs.
+   Show that for graphs and special goal sets there is no
    constant competitive exploration strategy. How can we adapt the
    LB construction for the non-approximation result for the optimal search path?
+   Goal set: vertices of a planar graph. Does a constant competitive
    approximation of the search ratio exists?

3.5:   Alternative cost measures: Dovetailling and Certificate paths approximations

+   Explain the multi-list traversal problem for
    known depth but unknown order.
+   Prove that for list a list L=(f_1,f_2,…,f_m)
    where $f_1 \geq f_2 \geq … \geq f_m$
    the optimal partially informed strategy has
    running time opt_L:= $\min_i (i \times f_i)$
+   Explain the hyperbolic dovetailing strategy for unknown list length.
+   Prove that the dovetailing strategy has ratio
    $$O(opt\_L \log(\min(m, opt\_L)))$$
+   Prove that the above ratio is optimal for any uninformed strategy.

+   Explain the definition of a certificate path in a simple polygon,
        starting from the kernel. Why do we want to use such a path?
+   What are the extreme cases for a start point in convex position?
+   What is the idea of the design of the spiral strategy for
        the approximation of the certificate paths?
+   How did we design the optimal spiral strategy?
+   Sketch the proof for the optimality of the spiral strategy
        against the certificate:
                a) Arbitrary strategy, register along n rays (2pi/n angular distance)
                b) Stop at a certain moment in time, consider minimal and maximal visits
                c) Define corresponding certificates
                d) Reorder the discrete strategy, this outperforms original strategy
                e) Optimisation of the sum of functionals

Chapter 4: Online exploration

4.1-4.2: Preliminaries and rectilinear polygons

+   What segments have to be visited for the online exploration of a simple polygon?
        Explain: Cuts, necessary cuts and essential cuts!
+   What is the main advantage in Rectilinear polygons?
                Explain the idea of the optimal offline computation!
                Explain the idea of the efficient online exploration!
+   Sketch the proof for the \sqrt(2)-approximation (L_2-metric) of the
        online path against the offline solution.
+   What results can be derived for the approximation of the optimal search path?

4.3: General simple polygons and looking around a corner

+   Explain the problem of looking around a corner.
        What kind of parameterisation did we use?
+   Analyse the half-circle strategy with radius 1/2 (with radius 1).
        Present the ratios!
+   Present the lower bound construction for looking around the
        corner. Analyse the two cases.
+   Present the main idea for the design of an optimal corner strategy.
                a) Define the requirements formally!
                b) Present the idea of the computation by incorporation
                        of the (unknown) optimal ratio.
+   Sketch the proof for the optimality of the corner strategy.
+   What is the main idea for the online exploration of a
        general simple polygon?
+   Give a simple lower bound for the online exploration task
        of a simple polygon.

+   Give an example that the idea of the rectilinear version
       (visit the vertices in the order along the boundary)
       will not end in a constant competitive strategy.
+   Apply the exploration strategy to the constant competitive
       approximation of the optimal search ratio.

4.4: Polygons with holes

+   Lower bound: Show that there is no constant competitive exploration
       strategy for polygons with holes.
       Sketch the lower bound construction and explain the analysis.
+   What is the consequence for the approximation of an optimal
       search path?
       Why do we have to adapt the construction for the non-approximation
       result of the search path?