Selected Topics in Algorithmics, SS15
Exercise Sheet “5”: Top-Down Sampling
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Wednesday 8th of July, 14:30 pm**. *There is a letterbox in front of room E.01 in the LBH building.*
- *You may work in groups of at most two participants.*

Exercise 12: Arrangement in 3 dimensions (6 Points)

Given a set N of n planes in three dimensions, the arrangement $G(N)$ formed by planes in N is a natural spacial partition of $\mathbb{R}^3$: the planes in N partitions $\mathbb{R}^3$ into $O(n^3)$ polyhedra, and $G(N)$ is the collection of those polyhedra together with their low dimensional faces. The canonical triangulation $H(N)$ for $G(N)$ is constructed as follows: for each polyhedron P of $G(N)$, we triangulate each facet F of P by linking all the vertices of F to its bottommost vertex, and then we link all vertices of P to its bottommost vertex. Actually, $H(N)$ partitions each polyhedron of $G(N)$ into tetrahedra. We make a general position assumption that any two planes must intersect exactly at a line, any three planes must intersect exactly at a point, and any four planes must not intersect. Please answer the following questions.

1. Due to the general position assumption, what is the maximum number of planes that define a tetrahedron in $H(N)$
2. Assume that we have already adopted the top-down sampling to build up a search structure for $G(N)$. What is the expected query time for the point location query in $G(N)$.
3. Assume that the construction of the search structure except the recursion takes expected $O(n^3)$ time. What is the total expected construction time?

Exercise 13: Ascent Structure for Planar Arrangement (4 Points)

We consider constructing the ascent structure during the top-down sampling for the planer arrangement of lines. Let N be a set of n lines in the plane, and let R be a random sample of N of size r, where r is a large enough constant. Let $G(N)$ be the arrangement of N, and let $H(N)$ be the canonical triangulation of $G(N)$. We assume that for each triangle $\triangle \in H(R)$, we already compute $G(N(\triangle)) \cap \triangle$, where $N(\triangle)$ denotes the set of lines in $N \setminus R$ that intersect $\triangle$, and $|N(\triangle)| = O(\frac{n}{r} \log r)$. Please explain how to associate each face of $G(N(\triangle)) \cap \triangle$ with a parent pointer to a face of $G(N)$. (Of course, we make a general position assumption that no two lines are parallel to each other, and no three lines intersect at the same point.)

Bonus 2: Flip Coins (5 points)

Assume we have a coin whose head probability is 3/4. Please use the well-known Chernoff bound to prove the following. (You do not need provide the exact numbers, but just give some inequalities.)

1. How many trials is it sufficient to ensure the probability that the number of heads is larger than the number of tails to be at least 95%?

2. How many trials is its sufficient to ensure the probability that the number of heads is double the number of tails to be 75%?