Institut für Informatik
Prof. Dr. Heiko Röglin
Clemens Rösner

Probabilistic Analysis
of Algorithms
Summer 2015

Problem Set 1

Problem 1

We flip a fair coin n times. For $k \in \mathbb{N}$, find an upper bound on the probability that there is a sequence of $\lceil \log_2 n \rceil + k$ consecutive heads. Your bound should decrease exponentially in k.

Problem 2

A monkey types on a 26-letter keyboard that has lowercase letters only. Each letter is chosen independently and uniformly at random from the alphabet. If the monkey types 1,000,000,000 letters, what is the expected number of times the sequence "banana" appears?

Problem 3

Each box of cereal contains one out of n possible coupons. Once you obtain all n coupons, you win a prize. Assuming that the coupon in each box is chosen independently and uniformly at random from the n possibilities, what is the expected number of boxes you have to buy before you can claim the prize?

Problem 4

In a quiz show 100 participants $P_1, \ldots, P_{100}$ can win a trip to Hawaii. The host of the show sets up a room with 100 closed boxes. Every box contains a sheet with a unique number from $\{1, 2, \ldots, 100\}$, and the boxes are placed one after another in a random order in the room. The participants, who can agree upfront on a strategy but cannot communicate anymore once the show has started, enter the room one after another.

The first participant P_1 enters the room first. He can open 50 of the boxes. If none of these boxes contains his own number 1, then all participants lose. Otherwise, he closes all the boxes again and leaves the room. He is not allowed to reorder the boxes or to leave any other hints. Then the second participant P_2 enters the room. He can also open 50 of the boxes. If none of these boxes contains his own number 2, then all participants lose. Otherwise, he closes all the boxes again and leaves the room. He is not allowed to reorder the boxes or to leave any other hints. This is continued until the last participant P_{100}.

The participants win if and only if each of them opens the box with his own number. If there is a single participant who does not open the box with his own number, everybody loses. Advise the participants on a strategy and compute the probability of winning according to your strategy.