Discrete and Computational Geometry, WS1415
Exercise Sheet “9”: Construction of Abstract Voronoi Diagrams
University of Bonn, Department of Computer Science I

- Written solutions have to be prepared until Tuesday 16th of December 14:00 pm. There will be a letterbox in the LBH building.
- You may work in groups of at most two participants.
- Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.
- If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom

Exercise 22: Randomized Incremental Algorithm for Abstract Voronoi Diagrams (History Graph) (4 Points)

Consider an admissible bisecting curve system \((S, J)\), and make a general position assumption that no four curves in \(J\) intersect at the same point. Let \(s_1, s_2, \ldots, s_n\) be a random sequence of \(S\), and let \(R^i = \{\infty, s_1, s_2, \ldots, s_i\}\). Please develop a randomized algorithm to construct the abstract Voronoi diagram \(V(S)\) by computing \(V(R^2), V(R^3), \ldots, V(R^n)\) iteratively using the history graph. In other words, for \(i \geq 2\), obtain \(V(R^{i+1})\) from \(V(R^i)\) by insertion \(s_{i+1}\). Let a configuration be a Voronoi edge of \(V(R^i)\), for \(2 \leq i \leq n\)

1. Define the parent and child relation between a configuration in \(V(R^i)\) \(\setminus V(R^{i+1})\) and a configuration in \(V(R^{i+1})\) \(\setminus V(R^i)\)

2. Please prove that if a site conflicts a configuration, there exists a path from the root of the history graph to the configuration along which all configuration is in conflict with the site.

3. Prove that the expected time complexity of inserting \(s^i\) is \(O(\log i)\)
Exercise 23: Removal of General Position Assumption (4 Points)

Consider an admissible bisecting curve system \((S, \mathcal{J})\) without the general position assumption that no four curves in \(\mathcal{J}\) intersect at the same point. In other words, more than three curves in \(\mathcal{J}\) can intersect at the same point, and the degree of a Voronoi vertex can be more than three. Please complete the following:

- Use a constant number of sites to define a Voronoi edge, i.e., formulate a configuration for a Voronoi edge. Note that a site can appear more than once in a configuration.

- Please describe how to update the conflict graph after inserting \(s\) into \(V(R)\).