Exercise 4: Triangulation (4 Points)

Given a set $N$ of $n$ points in the plane, a triangulation $T(N)$ of $N$ is a maximal planar straight-light graph, i.e., every edge is a straight-line segment, and no edge can be added to maintain the planarity. Let $S_1, S_2, \ldots, S_n$ be a random sequence of $N$, and let $N^i = \{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $T(N)$ by computing $T(N^3), T(N^4), \ldots, T(N^n)$ iteratively using the conflict lists. In other words, for $i \geq 3$, obtain $T(N^{i+1})$ from $T(N^i)$ by adding $S_{i+1}$. (Hint: Add three dummy points, $p_1, p_2,$ and $p_3$, in the infinity such that the outer boundary of $T(N^i \cup \{p_1, p_2, p_3\})$ is a triangle whose vertices are $p_1, p_2,$ and $p_3$ for $1 \leq i \leq n$.)

1. Describe the insertion of $S_{i+1}$

2. Define a conflict relation between a triangle in $T(N^i)$ (i.e., $T(N^i \cup \{p_1, p_2, p_3\})$) and a point in $N \setminus N^i$

3. Prove the expected cost of inserting $S_{i+1}$ to be $O\left(\frac{n}{i+1}\right)$ and the expected cost of construction $T(N)$ to be $O(n \log n)$
Exercise 5: Planar Convex Hull by Conflict Lists (4 Points)

Given a set $N$ of $n$ points in the plane, a convex hull $H(N)$ of $N$ is a minimal convex polygon containing $N$. Let $S_1, S_2, \ldots, S_n$ be a random sequence of $N$, and let $N^i$ be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^3), H(N^4), \ldots, H(N^n)$ iteratively using the conflict lists. In other words, for $i \geq 3$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding $S_{i+1}$.

1. Describe the insertion of $S_{i+1}$

2. Define a conflict relation between an edge of $H(N^i)$ and a point in $N \setminus N^i$

3. Prove the expected cost of inserting $S^{i+1}$ to be $O\left(\frac{n}{i+1}\right)$ and the expected cost of construction $H(N)$ to be $O(n \log n)$.

Exercise 6: Voronoi Diagrams (4 Points)

Given a set $S$ of $n$ points in the Euclidean plane, the Voronoi diagram $V(S)$ partitions the plane into Voronoi regions $\text{VR}(p, S)$, $p \in S$, such that all points in $\text{VR}(p, S)$ share the same nearest site $p$ among $S$. We make a general position assumption that no more than three points of $S$ are located on the same circle. Let $e$, $v$, and $u$ be the numbers of edges, vertices, unbounded faces of $V(S)$.

1. Please prove $e = 3(n - 1) - u$ and $v = 2(n - 1) - u$. (Hint: use Euler’s formula)

2. Please explain that if $u$ is fixed, the number of vertices will not increase without the general position assumption.