Exercise 28: Spanners and Closest Pairs (4 Points)

Let $S$ denote a finite point set in $\mathbb{R}^d$. Let $1 < t \leq 2$ and let $G = (S, E)$ be a $t$-spanner with vertex set $S$ and edge set $E$.

a) Show that for at least one closest pair $v, w$ in $S$ the edge $\{v, w\}$ belongs to $E$. Furthermore, if $t < 2$, this is even true for all closest pairs.

b) Let $p$ be a nearest neighbor of $q$ in $S$. Does this imply that $\{p, q\}$ belongs to $E$?

Exercise 29: WSPD and Centers (4 Points)

Prove or disprove the following statement: Two point sets $A, B$ with bounding box $R(A)$ and $R(B)$ are well-separated with parameter $s$, if and only if there are two circles $C_A$ and $C_B$ of some radius $r$, where $R(A) \subset C_A$, $R(B) \subset C_B$ and the distance between $C_A$ and $C_B$ is $\geq r \cdot s$, and the center of $C_A$ and of $C_B$ coincides with the center of the bounding box of $A$ and of $B$, respectively.
Exercise 30: WSPD 2-dimensional Example (4 Points)

Consider the point set $S \subset \mathbb{R}^2$ depicted twice below. Use the algorithm presented in the lecture to construct a WSPD of $S$, given the separation ratio $s = 1$.

Start with computing the split-tree, and draw the resulting bounding boxes. Use these bounding boxes to construct the WSPD. You may assume that the procedure FindPairs($v, w$) only verifies if the two point sets $S_v$ and $S_w$ are well separated with respect to circles, whose center points are located at the center of the corresponding bounding box.